

Relativistic Heavy-Ion Collisions within 3-Fluid Hydrodynamics*: Hadronic Scenario

Yu.B. Ivanov,^{1,2,†} V.N. Russkikh,^{1,2,‡} and V.D. Toneev^{1,3,§}

¹*Gesellschaft für Schwerionenforschung mbH, Planckstr. 1, D-64291 Darmstadt, Germany*

²*Kurchatov Institute, Kurchatov sq. 1, Moscow 123182, Russia*

³*Joint Institute for Nuclear Research, 141980 Dubna, Moscow Region, Russia*

A 3-fluid hydrodynamic model for simulating relativistic heavy-ion collisions is introduced. Alongside with two baryon-rich fluids, the new model considers time-delayed evolution of a third, baryon-free (i.e. with zero net baryonic charge) fluid of newly produced particles. Its evolution is delayed due to a formation time τ , during which the baryon-free fluid neither thermalizes nor interacts with the baryon-rich fluids. After the formation it starts to interact with the baryon-rich fluids and quickly gets thermalized. Within this model with pure hadronic equation of state, a systematic analysis of various observables at incident energies between few and about 160A GeV has been done as well as comparison with results of transport models. We have succeeded to reasonably reproduce a great body of experimental data in the incident energy range of $E_{\text{lab}} \simeq (1-160)A$ GeV. The list includes proton and pion rapidity distributions, proton transverse-mass spectra, rapidity distributions of Λ and $\bar{\Lambda}$ hyperons, elliptic flow of protons and pions (with the exception of proton v_2 at 40A GeV), multiplicities of pions, positive kaons, ϕ mesons, hyperons and antihyperons, including multi-strange particles. This agreement is achieved on the expense of substantial enhancement of the interflow friction as compared to that estimated proceeding from hadronic free cross sections. However, we have also found out certain problems. The calculated yield of K^- is approximately by a factor of 1.5 higher than that in the experiment. We have also failed to describe directed transverse flow of protons and pion at $E_{\text{lab}} \geq 40A$ GeV. This failure apparently indicates that the used EoS is too hard and thereby leaves room for a phase transition.

PACS numbers: 24.10.Nz, 25.75.-q

Keywords: relativistic heavy-ion collisions, hydrodynamics, equation of state

I. INTRODUCTION

Relativistic nucleus-nucleus collisions at incident energies $E_{\text{lab}} \simeq (10-40)A$ GeV presently attracts special attention, since the highest baryon densities [1, 2] and highest relative strangeness [3] at moderate temperatures are expected in this incident energy range. The interest to this energy range has been recently also revived in connection with new experimental results from low-energy scanning SPS program [4] and the project of the new accelerator facility SIS300 at GSI [5]. This domain of the nuclear phase diagram is less explored both experimentally and theoretically as compared to the corresponding extremes, i.e. cold compressed matter and baryon-free hot matter. Moreover, a critical point of the QCD phase diagram may occur to be accessible in these reactions [6, 7]. There are already available experimental data [4] pointing out that something interesting really happens in this energy range. In order to draw information on properties of hot and compressed nuclear matter from available experimental data, we need better understanding of dynamics of nucleus-nucleus collisions under investigation.

A direct way to address thermodynamic properties of the matter produced in these reactions consists in application of hydrodynamic simulations to nuclear collisions. However, finite nuclear stopping power, revealing itself at high incident energies, makes the collision dynamics non-equilibrium and prevents us from application of conventional hydrodynamics especially at the initial stage of the reaction. Since the resulting non-equilibrium is quite strong, introduction of viscosity and thermal conductivity does not help to overcome this difficulty, because by definition they are suitable for weak non-equilibrium. A possible way out is taking advantage of a multi-fluid approximation to heavy-ion collisions, pioneered by Los Alamos group [8, 9] and further developed at the Kurchatov Institute [10, 11, 12], Frankfurt [13, 14, 15] and GSI [16, 17]. The last development [16, 17], i.e. the mean-field dynamics in the multi-fluid background, concerns mostly moderate energies around 1 GeV/nucleon and therefore will not be discussed here.

The first extension of the original 2-fluid model [8, 9], the two-fluid hydrodynamics with free-streaming radiation of pions, was advanced in [10]. The initial stage of heavy-ion collisions definitely is a highly non-equilibrium process. Within the hydrodynamic approach this non-equilibrium is simulated by means of a 2-fluid approximation, which takes care of the finite stopping power of nuclear matter [18, 19], and simultaneously describes the entropy generation at the initial stage. The radiated pions form a baryon-free matter in the mid-rapidity region, while two baryon-rich fluids simulate the propa-

*<http://theory.gsi.de/~mfd/>

†e-mail: Y.Ivanov@gsi.de

‡e-mail: russ@ru.net

§e-mail: V.Toneev@gsi.de

gation of leading particles. The pions are the most abundant species of the baryon-free matter which may contain any hadronic, including baryon-antibaryon pairs, and/or quark-gluon species.

First applications of the 2-fluid model with direct pion radiation [11, 12] were quite successful for describing heavy-ion collisions in the wide range of incident energies, from SIS to SPS. In these 3D hydrodynamic simulations the whole process of the reaction was described, i.e. the evolution from the formation of a hot and dense nuclear system to its subsequent decay. This is in distinction to numerous other simulations, which treat only the expansion stage of a fireball formed in the course of the reaction, while the initial state of this dense and hot nuclear system is constructed from either kinetic simulations [20] or more general albeit model-dependent assumptions (e.g. see [21, 22]).

However, the approximation of free-streaming pions, produced in the mid-rapidity region, was still irritating from the theoretical point of view, in particular, because the relative momenta of the produced pions and the leading baryons are in the range of the Δ resonance for the incident energies considered. This would imply that the interaction between the produced pions and the baryon-rich fluids should be strong. The free-streaming assumption relies on a long formation time of produced pions. Indeed, the proper time for the formation of the produced particles is commonly assumed to be of the order 1 fm/c in the comoving frame. Since the main part of the produced pions is quite relativistic at high incident energies, their formation time should be long enough in the reference frame of calculation to prevent them from interacting with the baryon-rich fluids. However, this argument is qualitative rather than quantitative, and hence requires further verification. The first attempt to do this was undertaken by the Frankfurt group [13], which started to explore an opposite extreme. They assumed that the produced pions immediately thermalize, forming a baryon-free fluid (or a “fireball” fluid, in terms of [13]), and interact with the baryon-rich fluids. No formation time was allowed, and the strength of the corresponding interaction was guessed rather than microscopically estimated. This opposite extreme, referred to as a (2+1)-fluid model and being not quite justified either, yielded results substantially different from those of the free-streaming approximation. This was one of the reasons why in subsequent applications the Frankfurt group neglected the interaction between baryon-free and baryon-rich fluids while keeping the produced pions thermalized [14, 15], thus effectively restoring the free-streaming approximation. However, the assumed immediate thermalization of the fireball fluid together with the lack of interaction with baryon-rich fluids still was not a consistent approximation.

In the present paper we would like to introduce an extension of the multi-fluid approach for simulating heavy-ion collisions, i.e. a 3-fluid model with formation time. This model is a straightforward extension of the 2-fluid

model with radiation of direct pions [10, 11, 12] and (2+1)-fluid model [13, 14, 15]. We extend the above models in such a way that the created baryon-free fluid (which we call a “fireball” fluid, according to the Frankfurt group) is treated on equal footing with the baryon-rich ones. This implies that we allow a certain formation time for the fireball fluid, during which the matter of the fluid propagates without interactions. Moreover, we assume that the fireball matter gets quickly thermalized after its formation. The latter approximation is an enforced one, since we deal with the hydrodynamics rather than with kinetics. The interaction between fireball and baryon-rich fluids is estimated based on elementary cross sections. A brief account of this model has been already reported in Ref. [23].

The formation time (τ) is a conventional tool of the hadronic physics, which is associated with a finite time of string formation. It is incorporated in kinetic transport models such as UrQMD [24] and HSD [25]. In dense medium the separate strings can already interact, this is the reason of introduction of junctions and formation of color ropes. In fact, this string interaction is a method of extending the treatment beyond the approximation of binary collisions which is inherent to the conventional Boltzmann equation. This interaction does not invalidate the concept of formation time but rather extends it to the case of multi-particle collisions. The concept of strings becomes irrelevant in the deconfined state of matter. Here there are two points of view: (i) quarks and gluons are instantly produced according to the QCD perturbation theory, or (ii) they are mediated by a coherent color field [26, 27], i.e. first the coherent color field is produced which subsequently decays into incoherent fluctuations – quarks and gluons. The first mechanism implies that $\tau=0$, while the second one still makes room for a finite τ . Therefore, the fitted value of τ may help to distinguish between these two mechanisms of the QGP production.

The developed code allows calculations with various equations of states (EoS), which enter as a separate block of the code. We have started with the simplest, purely hadronic EoS which involves only a simple density dependent mean field providing saturation of cold nuclear matter at normal nuclear density $n_0 = 0.15 \text{ fm}^{-3}$ and with the proper binding energy -16 MeV. This EoS is a natural reference point for any other more elaborate EoS. Much to our surprise, this trivial EoS turned out to be able to reasonably reproduce a great body of experimental data. Taking advantage of the modern computers, substantial work has been also done on improvement of numerics of the model. These results are reported in this paper.

II. 3-FLUID HYDRODYNAMIC MODEL WITH DELAYED FORMATION

The derivation of equations of the 3-fluid model presented below is in fact only an illustration of physical assumptions which this model is based on. Indeed, the Boltzmann equation, from which this derivation starts, strictly speaking is inapplicable to a dense and strongly interacting matter. Therefore, the applicability of the 3-fluid model to heavy-ion collisions is certainly an assumption which should be verified in comparison with experimental data. However, this is a general situation with dense systems even in simpler cases. The basic reason for introduction of the 3-fluid approximation is simulation of the finite stopping power which is important at the formation stage of the initial hot and dense blob of nuclear matter. In this sense, it is an alternative to constructing this initial blob by means of either various kind kinetic transport models [20] or model assumptions [21, 22].

A. Basic Formulation

Unlike the conventional hydrodynamics, where local instantaneous stopping of projectile and target matter is assumed, a specific feature of the dynamic 3-fluid description is a finite stopping power resulting in a counter-streaming regime of leading baryon-rich matter. Experimental rapidity distributions in nucleus-nucleus collisions support this counter-streaming behavior, which can be observed for incident energies between few and 200A GeV. The basic idea of a 3-fluid approximation to heavy-ion collisions [10, 28] is that at each space-time point $x = (t, \mathbf{x})$ the generally nonequilibrium distribution function of baryon-rich matter, $f_{\text{br}}(x, p)$, can be represented as a sum of two distinct contributions

$$f_{\text{br}}(x, p) = f_{\text{p}}(x, p) + f_{\text{t}}(x, p), \quad (1)$$

initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles, populating the mid-rapidity region, are associated with a fireball (f) fluid described by the distribution function $f_{\text{f}}(x, p)$. Therefore, the 3-fluid approximation is a minimal way to simulate the finite stopping power at high incident energies. Note that both the baryon-rich and fireball fluids may consist of any type of hadrons and/or partons (quarks and gluons), rather than only nucleons and pions. However, here and below we suppress the species label at the distribution functions for the sake of transparency of the equations.

With the above-introduced distribution functions f_{α} ($\alpha = \text{p, t, f}$), the coupled set of relativistic Boltzmann equations looks as follows:

$$p_{\mu} \partial_x^{\mu} f_{\text{p}}(x, p) = C_{\text{p}}(f_{\text{p}}, f_{\text{t}}) + C_{\text{p}}(f_{\text{p}}, f_{\text{f}}), \quad (2)$$

$$p_{\mu} \partial_x^{\mu} f_{\text{t}}(x, p) = C_{\text{t}}(f_{\text{p}}, f_{\text{t}}) + C_{\text{t}}(f_{\text{t}}, f_{\text{f}}), \quad (3)$$

$$p_{\mu} \partial_x^{\mu} f_{\text{f}}(x, p) = C_{\text{f}}(f_{\text{p}}, f_{\text{t}}) + C_{\text{f}}(f_{\text{p}}, f_{\text{f}}) + C_{\text{f}}(f_{\text{t}}, f_{\text{f}}), \quad (4)$$

where C_{α} denote collision terms between the constituents of the three fluids. We have omitted intra-fluid collision terms, like $C_{\text{p}}(f_{\text{p}}, f_{\text{p}})$, since below they will be canceled anyway. The displayed inter-fluid collision terms have a clear physical meaning: $C_{\text{p/t}}(f_{\text{p}}, f_{\text{t}})$, $C_{\text{p/t}}(f_{\text{p/t}}, f_{\text{f}})$, and $C_{\text{f}}(f_{\text{p/t}}, f_{\text{f}})$ give rise to friction between p-, t- and f-fluids, and the term $C_{\text{f}}(f_{\text{p}}, f_{\text{t}})$ takes care of particle production in the mid-rapidity region. Note that up to now we have done no approximation, except for hiding intra-fluid collision terms.

Let us proceed to approximations which justify the term “fluids” having been used already. We assume that constituents within each fluid are locally equilibrated, both thermodynamically and chemically, i.e. that f_{α} are equilibrium distributions. In particular, this implies that the intra-fluid collision terms are indeed zero. This assumption relies on the fact that intra-fluid collisions are much more efficient in driving a system to equilibrium than inter-fluid interactions. As applied to the fireball fluid, this assumption requires some additional comments, related to the concept of a finite formation time. During the proper formation time τ after production, the fireball fluid propagates freely, interacting neither with itself nor with the baryon-rich fluids. After this time interval, the fireball matter starts to interact with both itself and the baryon-rich fluids and, as a result, locally thermalizes. Being heated up, these three fluids may contain not only hadronic and but also deconfined quark-gluon species, depending on the EoS used.

The above assumption suggests that interaction between different fluids should be treated dynamically. To obtain the required dynamic equations, we first integrate the kinetic Eqs. (2)–(4) over momentum and sum over particle species with weight of baryon charge. This way we arrive to equations of the baryon charge conservation

$$\partial_{\mu} J_{\alpha}^{\mu}(x) = 0, \quad (5)$$

for $\alpha = \text{p}$ and t , where $J_{\alpha}^{\mu} = n_{\alpha} u_{\alpha}^{\mu}$ is the baryon current defined in terms of baryon density n_{α} and hydrodynamic 4-velocity u_{α}^{μ} normalized as $u_{\alpha\mu} u_{\alpha}^{\mu} = 1$. Eq. (5) implies that there is no baryon-charge exchange between p- and t-fluids, as well as that the baryon current of the fireball fluid is identically zero, $J_{\text{f}}^{\mu} = 0$. Integrating kinetic Eqs. (2)–(4) over momentum with weight of 4-momentum p^{ν} and summing over all particle species, we arrive at equations of the energy-momentum exchange for energy-momentum tensors $T_{\alpha}^{\mu\nu}$ of the fluids

$$\partial_{\mu} T_{\text{p}}^{\mu\nu}(x) = -F_{\text{p}}^{\nu}(x) + F_{\text{fp}}^{\nu}(x), \quad (6)$$

$$\partial_{\mu} T_{\text{t}}^{\mu\nu}(x) = -F_{\text{t}}^{\nu}(x) + F_{\text{ft}}^{\nu}(x), \quad (7)$$

$$\partial_{\mu} T_{\text{f}}^{\mu\nu}(x) = F_{\text{p}}^{\nu}(x) + F_{\text{t}}^{\nu}(x) - F_{\text{fp}}^{\nu}(x) - F_{\text{ft}}^{\nu}(x), \quad (8)$$

where the F_{α}^{ν} are friction forces originating from inter-fluid collision terms in the kinetic Eqs. (2)–(4). F_{p}^{ν} and F_{t}^{ν} in Eqs. (6)–(7) describe energy-momentum loss of baryon-rich fluids due to their mutual friction. A part of this loss $|F_{\text{p}}^{\nu} - F_{\text{t}}^{\nu}|$ is transformed into thermal excitation of these fluids, while another part $(F_{\text{p}}^{\nu} + F_{\text{t}}^{\nu})$ gives rise

to particle production into the fireball fluid (see Eq. (8)). F_{fp}^ν and F_{ft}^ν are associated with friction of the fireball fluid with the p- and t-fluids, respectively. Note that Eqs. (6)–(8) satisfy the total energy–momentum conservation

$$\partial_\mu (T_{\text{p}}^{\mu\nu} + T_{\text{t}}^{\mu\nu} + T_{\text{f}}^{\mu\nu}) = 0. \quad (9)$$

As described above, the energy–momentum tensors of the baryon-rich fluids ($\alpha = \text{p}$ and t) take the conventional hydrodynamic form

$$T_\alpha^{\mu\nu} = (\varepsilon_\alpha + P_\alpha) u_\alpha^\mu u_\alpha^\nu - g^{\mu\nu} P_\alpha \quad (10)$$

in terms of the proper energy density, ε_α , and pressure, P_α . For the fireball, however, only the energy–momentum tensor of the formed matter is of practical interest for us. Since we treat the fireball matter as a fluid, we have nothing to do but assume that it is formed already thermalized. This assumption implies that its thermalization time is essentially shorter than its formation time. Moreover, this assumption is in the spirit of other assumptions made: we distinguish the main nonequilibrium associated with finite stopping power and consider all the rest of dynamics within local equilibrium. Thus, only the formed (and by assumption thermalized) part of the energy–momentum tensor is described by this hydrodynamic form

$$T_{\text{f}}^{(\text{eq})\mu\nu} = (\varepsilon_{\text{f}} + P_{\text{f}}) u_{\text{f}}^\mu u_{\text{f}}^\nu - g^{\mu\nu} P_{\text{f}}. \quad (11)$$

Its evolution is defined by an Euler equation with a retarded source term

$$\partial_\mu T_{\text{f}}^{(\text{eq})\mu\nu}(x) = -F_{\text{fp}}^\nu(x) - F_{\text{ft}}^\nu(x) + \int d^4x' \delta^4(x - x' - U_F(x')\tau) [F_{\text{p}}^\nu(x') + F_{\text{t}}^\nu(x')], \quad (12)$$

where τ is the formation time, and

$$U_F^\nu(x') = \frac{u_{\text{p}}^\nu(x') + u_{\text{t}}^\nu(x')}{|u_{\text{p}}(x') + u_{\text{t}}(x')|} \quad (13)$$

is a free-propagating 4-velocity of the produced fireball matter, which is evidently a time-like 4-vector. In fact, this is the velocity at the moment of production of the fireball matter. According to Eq. (12), the energy and momentum of this matter appear as a source in the Euler equation only later, i.e. after the time interval $U_F^0\tau$ upon the production, and in different space point $\mathbf{x}' - \mathbf{U}_F(x')\tau$, as compared to the production point \mathbf{x}' . From the first glance, one can immediately simplify the r.h.s. of Eq. (12) by performing integration with the δ -function. However, this integration is not so straightforward, since the expression under the δ -function, $x - x' - U_F(x')\tau = 0$, may have more than one solution with respect to x' . The latter would mean that the matter produced in several different space-time points x' is simultaneously formed in the same space-time point x . This is possible due to the nonlinearity of the hydrodynamic equations.

The above discussed free-propagating fireball matter is, of course, a certain approximation for treating free-streaming particles, which this matter consists of. These produced free-streaming particles are characterized by a distribution function, $f(p)$, in the momentum space, which is determined by their production cross sections, e.g. $p^0 d\sigma_{NN \rightarrow \pi X} / d^3p$. In particular, this means that particles get formed at different time instants $\tau_{\text{particle}} \gamma_{\text{particle}}$, depending on the particle velocity (γ_{particle} is the gamma factor of the particle). This particle formation is governed by a particle formation time τ_{particle} which differs from our τ we use in the hydrodynamic formulation. Our τ has a meaning of the particle formation time τ_{particle} averaged over the distribution $f(p)$:

$$\tau \gamma_{\text{f}} = \int \frac{d^3p}{p^0} \tau_{\text{particle}} \gamma_{\text{particle}} f(p) / \int \frac{d^3p}{p^0} f(p), \quad (14)$$

where γ_{f} is the gamma factor of the unformed fireball matter. Eq. (14) shows that τ is always longer than τ_{particle} . In particular, if the fireball matter consists of pions, which have approximately thermal distribution with temperature ≈ 100 MeV, then $\tau \approx 2\tau_{\text{particle}}$.

The free-streaming particles are not only formed at different time instants but also in different space points because of the same $f(p)$ distribution. Thus, another approximation we have done consists in neglecting this spacial spread. We could take it into account by changing the δ -function in the r.h.s. of Eq. (12) to a smooth distribution of formed particles. This would result in spatial smearing of formation of the fireball fluid. However, at the same time it would make the model formulation more complex, since this smooth distribution of formed particles should depend on the initial $f(p)$ distribution and, hence, on the cross sections, like $p^0 d\sigma_{NN \rightarrow \pi X} / d^3p$. Moreover, it would be another source of uncertainty, since the $NN \rightarrow \pi X$ process is not the only relevant one. Therefore, for the first estimate of effects of the formation time we avoid these complications.

The residual part of $T_{\text{f}}^{\mu\nu}$ (the free-propagating one) is defined as

$$T_{\text{f}}^{(\text{fp})\mu\nu} = T_{\text{f}}^{\mu\nu} - T_{\text{f}}^{(\text{eq})\mu\nu}. \quad (15)$$

The equation for $T_{\text{f}}^{(\text{fp})\mu\nu}$ can be easily obtained by taking the difference between Eqs. (8) and (12). If all the fireball matter turns out to be formed before freeze-out, then this equation is not needed. Thus, the 3-fluid model introduced here contains both the original 2-fluid model with pion radiation [10, 11] and the (2+1)-fluid model [13, 14, 15] as limiting cases for $\tau \rightarrow \infty$ and $\tau = 0$, respectively.

B. Friction between Baryon-Rich Fluids and their Unification

The nucleon–nucleon cross sections at high energies are strongly forward–backward peaked. This fact, which

originally served as justification for subdividing baryonic matter into target and projectile fluids, was used in [18] to estimate the friction forces, F_p^ν and F_t^ν , proceeding from only NN elastic scattering. Later these friction forces were calculated [29] based on (both elastic and inelastic) experimental inclusive proton-proton cross sections. In the present calculations we use the following form of the projectile-target friction

$$F_\alpha^\nu = \vartheta^2 \rho_p^\xi \rho_t^\xi [(u_\alpha^\nu - u_\alpha^\nu) D_P + (u_p^\nu + u_t^\nu) D_E], \quad (16)$$

$\alpha = p$ or t , $\bar{p} = t$ and $\bar{t} = p$. Here, ρ_α^ξ denotes a kind of "scalar densities" of the p - and t -fluids (see details below),

$$D_{P/E} = m_N V_{\text{rel}}^{\text{pt}} \sigma_{P/E}(s_{\text{pt}}), \quad (17)$$

where m_N is the nucleon mass, $s_{\text{pt}} = m_N^2 (u_p^\nu + u_t^\nu)^2$ is the mean invariant energy squared of two colliding nucleons from the p - and t -fluids,

$$V_{\text{rel}}^{\text{pt}} = [s_{\text{pt}}(s_{\text{pt}} - 4m_N^2)]^{1/2} / 2m_N^2 \quad (18)$$

is the mean relative velocity of the p - and t -fluids, and $\sigma_{P/E}(s_{\text{pt}})$ are determined in terms of nucleon-nucleon cross sections integrated with certain weights (see [10, 11, 29] for details):

$$\sigma_P(s_{\text{pt}}) = \int_{\theta_{\text{cm}} < \pi/2} d\sigma_{NN \rightarrow NX} \left(1 - \cos \theta_{\text{cm}} \frac{p_{\text{out}}}{p_{\text{in}}} \right), \quad (19)$$

$$\sigma_E(s_{\text{pt}}) = \int_{\theta_{\text{cm}} < \pi/2} d\sigma_{NN \rightarrow NX} \left(1 - \frac{E_{\text{out}}}{E_{\text{in}}} \right). \quad (20)$$

Here the integration is restricted to the forward hemisphere ($\theta_{\text{cm}} < \pi/2$) of the center-of-mass scattering angles θ_{cm} , $p_{\text{in}} = (s_{\text{pt}}/4 - m_N^2)^{1/2}$ and $E_{\text{in}} = s_{\text{pt}}^{1/2}/2$ are the in-coming momentum and energy of the nucleon before the scattering in the NN c.m. frame, respectively, and p_{out} and E_{out} are the corresponding out-coming quantities. $\sigma_P(s_{\text{pt}})$ is a kind of a transport cross section, which is nonzero at any physical s_{pt} , as it is seen from Eq. (19). At the same time, the $\sigma_E(s_{\text{pt}})$ quantity, which is responsible for the fireball production, is zero for s_{pt} below the inelastic threshold. In particular, the latter feature makes $D_E = 0$, and hence the friction $F_\alpha = 0$, at vanishing relative velocity $u_p - u_t = 0$. The overall ϑ^2 factor in Eq. (16) is associated with unification of p - and t -fluid into a single one (see Eq. (23)), when their relative velocity gets small enough.

The above friction (16) is a certain extension of that derived in [29] and used in [10, 11, 12, 13, 14]. This original derivation was performed under assumption that baryon-rich fluids consist of only nucleons, or baryons, assuming that baryon-baryon cross sections are similar to proton-proton ones. Therefore, in the original expression for F_α^ν [10, 11, 12, 13, 14, 29] modified scalar densities of the p - and t -fluids are substituted by corresponding baryon densities (with $\xi_h = \xi_q = 1$ and $\vartheta^2 = 1$, see below). However, this original calculation is incomplete,

because it does not take into account :

- (i) various mesonic (and maybe, quark and gluonic) species produced in the collision
- (ii) possible multiparticle interactions which are quite probable in the dense medium,
- (iii) possible medium modifications of cross sections and effective masses, and
- (iv) quark and gluon interactions, if the system happens to undergo the phase transition to the quark-gluon phase.
- (v) Moreover, even experimental cross sections between hadronic species are only poorly known. As for the non-perturbative quark-gluon phase, this is the range of pure speculations.

In view of theses uncertainties, it is reasonable to make provision for tuning the above friction. For this purpose we introduced tuning factors $\xi(s_{\text{pt}})$ in the scalar densities of the p - and t -fluids

$$\rho_\alpha^\xi(s_{\text{pt}}) = \left(\rho_\alpha^{\text{bar.}} + \frac{2}{3} \rho_\alpha^{\text{mes.}} \right) \xi_h(s_{\text{pt}}) + \frac{1}{3} (\rho_\alpha^q + \rho_\alpha^g) \xi_q(s_{\text{pt}}), \quad (21)$$

where $\rho_\alpha^{\text{bar.}}$, $\rho_\alpha^{\text{mes.}}$, ρ_α^q and ρ_α^g are scalar densities of all baryons, all mesons, quarks and gluons, respectively, defined in the conventional way, i.e.

$$\rho_a(x) = m_a \int \frac{d^3p}{p_0} f_a(x, p) \quad (22)$$

for the a species of mass m_a which are $\rho_\alpha(T(x), \mu_B(x))$ in the thermodynamic limit. These scalar densities (but of course, only nucleon ones) result from the original derivation of Ref. [29]. We just extended the recipe of Ref. [29] to other species. In fact, in the nonrelativistic case the scalar density is identical to the usual particle-number density, since the mass m_a in Eq. (22) is canceled by p_0 in the denominator. Moreover, in the relativistic case the scalar density is a natural result, since it is a Lorentz invariant contrary to the particle-number density, which is the 4th component of a 4-vector. However, in the relativistic case we run into trouble with gluons and quarks. If one uses current masses for gluons and quarks, the corresponding scalar densities are either identically zero (for gluons) or negligibly small (for u and d quarks). To overcome this problem we imply that either thermal effective masses or constituent masses, depending on particular model, should be used in the scalar densities, or they should be calculated beyond the quasiparticle approximation (e.g. on the lattice).

Factors like $2/3$ and $1/3$ in Eq. (21) take into account the assumed scaling of cross sections in accordance with the naive valence-quark counting. Note that the original free cross section in (16) was estimated for the proton-proton pair. In Eq. (21) different tuning factors are introduced for hadronic and quark-gluon phase: ξ_h and ξ_q , respectively. If the system occurs in pure hadronic phase, ξ_h^2 directly scales the cross sections $\sigma_{P/E}$. If the system happens to be in a mixed phase, like in the cross-over phase transition, both ξ_h and ξ_q participate in the

cross-section scaling. Note that originally the scalar densities depend only on x , cf. Eq. (21). The introduced s -dependent tuning factors make the tuned scalar densities ρ_α^ξ also s -dependent. In general, ξ_h and ξ_q could be also assumed to be temperature dependent, since the $\sigma_{P/E}$ are obtained by averaging over certain thermal distributions. Nevertheless, we avoid to do this in order to keep the number of “fitting degrees of freedom” as low as possible.

When the baryon-rich fluids get decelerated enough, it is reasonable to unify them into a single fluid. This limit is not automatically present in the model formulation and therefore should be stipulated [106]. For this purpose we introduce an auxiliary function

$$\vartheta = 1 - \exp[-(V_{\text{rel}}^{\text{pt}}/\Delta V)^4] \quad (23)$$

where $V_{\text{rel}}^{\text{pt}}$ is the mean relative velocity of the p- and t-fluids of Eq. (18), and ΔV is the characteristic velocity of particles inside the flow (the Fermi velocity of nucleons in cold matter or the thermal velocity of particles in hot matter)

$$[1 - (\Delta V)^2]^{-1/2} - 1 = \max_{p,t} \left\{ \frac{\varepsilon_F}{m_N}, \frac{3T}{2m_N} \right\}, \quad (24)$$

where ε_F is the Fermi energy corresponding to the baryon density of the fluid, T is temperature of the fluid, and the maximum value is searched over quantities corresponding to the two overlapped (p- and t-) fluids. Thus, at $V_{\text{rel}}^{\text{pt}} \gg \Delta V$, $\vartheta = 1$ and the purely two-fluid regime is realized, while at $V_{\text{rel}}^{\text{pt}} \ll \Delta V$, $\vartheta = 0$ and the one-fluid regime takes place, which implies complete equilibration of the overlapping fluids. During the numeric simulation the two-fluid and one-fluid solutions are mixed in proportion ϑ and $1 - \vartheta$, respectively, thus providing a smooth approach to the one-fluid limit. This interpolation procedure concerns the pressure P_α and hydrodynamic 3-velocity \mathbf{v}_α , i.e. those quantities that are required for the hydrodynamic transport and are non-additive:

$$\tilde{P}_\alpha = \vartheta P_\alpha + (1 - \vartheta) \frac{n_\alpha}{n_{\text{tot}}} P_{\text{tot}}, \quad (25)$$

$$\tilde{\mathbf{v}}_\alpha = \vartheta \mathbf{v}_\alpha + (1 - \vartheta) \mathbf{v}_{\text{tot}}. \quad (26)$$

Here \tilde{P}_α and $\tilde{\mathbf{v}}_\alpha$ are interpolated quantities for the α fluid, and P_{tot} , \mathbf{v}_{tot} , n_{tot} , etc. are quantities derived from the total baryon density $J_{\text{tot}}^0 = J_p^0 + J_t^0$ and the 0ν components of the total baryon-rich energy-momentum tensor $T_{\text{tot}}^{0\nu} = T_p^{0\nu} + T_t^{0\nu}$, assuming that p- and t-fluids are unified. The difference between P_α and P_{tot} , and between \mathbf{v}_α and \mathbf{v}_{tot} , respectively, is as follows. From the solution of hydrodynamic equations (5)–(7) and (12) at each time step we obtain hydrodynamic quantities J_α^0 , $T_\alpha^{0\nu}$ and $T_f^{(\text{eq})0\nu}$. At the same time, we need, in particular, the pressure and hydrodynamic 4-velocity in order to proceed to the next time step. We have to calculate them based on the above hydrodynamic quantities and

the EoS. Here we can proceed in two ways. The conventional way of the multi-fluid hydrodynamics consists in calculating the pressure and 4-velocity for each fluid, i.e. solely based on the J_α^0 and $T_\alpha^{0\nu}$ quantities related to this fluid. This way we arrive at P_α and \mathbf{v}_α , which completely preserve the multi-fluid character of the solution. Alternatively, we can completely abandon the multi-fluid nature and assume that we deal with a locally equilibrium piece of baryon-rich matter, which is characterized by J_{tot}^0 and $T_{\text{tot}}^{0\nu}$, and then calculate the pressure and 4-velocity of this “unified” piece. Thereby we arrive at P_{tot} and \mathbf{v}_{tot} , which describe unified baryon-rich fluids as if they are mutually stopped.

This unification procedure was first proposed by the Los-Alamos group [8] and then used in subsequent applications of the multi-fluid dynamics [11, 12, 13, 14]. Evidently, this procedure also affects the stopping power and therefore can be added to the above list of uncertainties associated with the friction force F_α^ν . Other criteria of unification may produce different observable stopping, as it was shown in Ref. [15]. However, we prefer to keep the original criterion, keeping in mind that this is an important part of the stopping prescription.

As compared to the already conventional unification procedure described above, we have also introduced ϑ^2 factor in the friction force itself, cf. Eq. (16). It is justified, since only the ϑ fraction of each baryon-rich fluid remains in the two-fluid regime, as it follows from the above discussion. Therefore, in order to be consistent with the above unification procedure, we should keep only this ϑ fraction of the density of each fluid, ρ_α^ξ , in the friction term.

As it has been already mentioned, the way, in which the unification procedure is realized, affects the stopping of the nuclear matter. In fact, it is possible to avoid this artificial unification procedure. As it was shown in Refs. [16, 28], the formulation of the multi-fluid dynamics in terms of mean fields rather than of the EoS results in automatic unification of mutually stopped matter. However, the problem of such mean-field formulation is that the mean fields become unrealistically strong at high relative velocities of counter-streaming matter, whereas they should die out due to momentum dependence of self energies, as it was advocated in the HSD model Ref. [25]. Therefore, for the domain of high incident energies the present approach, based on a separate EoS in each fluid and complemented by the unification procedure, is certainly preferable as compared to the pure mean-field formulation of Refs. [16, 28]. While the latter formulation is definitely advantageous at moderate incident energies of the order of 1A GeV.

Eqs. (5)–(7) and (11), supplemented by a certain EoS and expressions for friction forces F^ν , form a full set of equations of the relativistic 3-fluid hydrodynamic model. The only quantity, we still need to define in terms of hydrodynamic variables and some cross sections, is the friction of the fireball fluid with the p- and t-fluids, F_{fp}^ν and F_{ft}^ν .

C. Interaction between Fireball and Baryon-Rich Fluids

Our aim here is to estimate the scale of the friction force between the fireball and baryon-rich fluids, similar to that done before for baryon-rich fluids [29]. To this end, we consider a simplified system, where all baryon-rich fluids consist only of nucleons, as the most abundant component of these fluids, and the fireball fluid contains only pions.

For incident energies from 10A (AGS) to 200A GeV (SPS), the relative nucleon-pion energies are in the resonance range dominated by the Δ -resonance. To estimate this relative energy we consider a produced pion, being at rest in the center of mass of the colliding nuclei, $p = \{m_\pi, 0, 0, 0\}_{cm}$. Baryon-rich fluids decelerate each other during their inter-penetration. This means that the nucleon momentum q should be smaller than the incident momentum, $q_0 = E_N < m_N \gamma_{cm}$, where γ_{cm} is the gamma factor of the incident nucleon in the c.m. frame. Calculating the invariant relative energy squared $s = (p + q)^2$ at $E_{lab} = 158A$ GeV, we obtain $s^{1/2} < 1.8$ GeV. This range of s precisely covers the resonance region, $1.1 \text{ GeV} < s^{1/2} < 1.8 \text{ GeV}$ [30]. At $E_{lab} = 10A$ GeV we arrive at $s^{1/2} < 1.3$ GeV, which is also within the resonance region. At even lower incident energies the strength of the fireball fluid becomes so insignificant, as compared with thermal mesons in the p- and t-fluids, that the way of treatment of its interaction with the baryon-rich fluids does not essentially affect the observables. For the same reason we do not apply any special prescription for the unification of the fireball fluid with the baryon-rich fluids, since this may happen only at relatively low incident energies $E_{lab} < 10A$ GeV.

The resonance-dominated interaction implies that the essential process is absorption of a fireball pion by a p- or t-fluid nucleon with formation of an R -resonance (most probably Δ). This produced R -resonance still belongs to the original p- or t-fluid, since its recoil due to absorption of a light pion is small. Subsequently this R -resonance decays into a nucleon and a pion already belonging to the original p- or t-fluid. Symbolically, this mechanism can be expressed as

$$N^\alpha + \pi^f \rightarrow R^\alpha \rightarrow N^\alpha + \pi^\alpha.$$

As a consequence, only the loss term contributes to the kinetic equation for the fireball fluid.

Proceeding from the above consideration, we write down the collision term between fireball-fluid pions and α -fluid nucleons ($\alpha = p$ or t) as follows

$$C_f(f_\alpha, f_f) = - \int \frac{d^3 q}{q_0} W^{N\pi \rightarrow R}(s) f_f^{(eq)}(p) f_\alpha(q), \quad (27)$$

where $s = (p + q)^2$,

$$W^{N\pi \rightarrow R}(s) = \frac{1}{2} \sqrt{(s - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2} \sigma_{tot}^{N\pi \rightarrow X}(s)$$

is the rate to produce a baryon R -resonance, and $\sigma_{tot}^{N\pi \rightarrow X}(s)$ is the parameterization of experimental pion-nucleon cross-sections [30]. Here, only the distribution function of formed (and hence thermalized) fireball pions, $f_f^{(eq)}$, enters the collision term, since the non-formed particles do not participate in the interaction by assumption.

Multiplying $C_f(f_\alpha, f_f)$ by the 4-momentum p^ν and integrating the result over momentum, we arrive at

$$\begin{aligned} F_{f\alpha}^\nu(x) &= \int \frac{d^3 q}{q_0} \frac{d^3 p}{p_0} p^\nu W^{N\pi \rightarrow R}(s) f_f^{(eq)}(p) f_\alpha(q) \\ &\simeq \frac{W^{N\pi \rightarrow R}(s_{f\alpha})}{m_\pi u_f^0} \left(\int \frac{d^3 q}{q_0} f_\alpha(q) \right) \left(\int \frac{d^3 p}{p_0} p^0 p^\nu f_f^{(eq)}(p) \right) \\ &= D_{f\alpha} \frac{T_f^{(eq)0\nu}}{u_f^0} \rho_\alpha, \end{aligned} \quad (28)$$

where we substituted p^0 and s by their mean values, $\langle p^0 \rangle = m_\pi u_f^0$ and $s_{f\alpha} = (m_\pi u_f + m_N u_\alpha)^2$, and introduced the transport coefficient

$$D_{f\alpha} = \frac{W^{N\pi \rightarrow R}(s_{f\alpha})}{m_N m_\pi} = V_{rel}^{f\alpha} \sigma_{tot}^{N\pi \rightarrow R}(s_{f\alpha}). \quad (29)$$

Here, $V_{rel}^{f\alpha} = [(s_{f\alpha} - m_N^2 - m_\pi^2)^2 - 4m_N^2 m_\pi^2]^{1/2} / (2m_N m_\pi)$ denotes the mean invariant relative velocity between the fireball and the α -fluids. Thus, we have expressed the friction $F_{f\alpha}^\nu$ in terms of the fireball-fluid energy-momentum density $T_f^{0\nu}$ (of only pions as yet), the scalar density ρ_α of the α -fluid (of only nucleons as yet), and a transport coefficient $D_{f\alpha}$. Note that this friction is zero until the fireball pions are formed, since $T_f^{(eq)0\nu} = 0$ during the formation time τ .

In fact, the above treatment is an estimate of the friction terms rather than their strict derivation. All the uncertainties mentioned in the previous subsection are well applied to the case under consideration. This peculiar way of evaluation is motivated by the form of the final result (28). An advantage of this form is that m_π and any other mass do not appear explicitly, and hence it allows a natural extension to any content of the fluid, including deconfined quarks and gluons, assuming that $D_{f\alpha}$ represents just a scale of the transport coefficient. Performing such extension, we assume that $T_f^{0\nu}$ represents the total energy-momentum density of the fireball-fluid and ρ_α is the total scalar density of the α -fluid, i.e. $\rho_\alpha \equiv \rho_\alpha^{\xi=1}$ in terms of Eq. (21). Here we have omitted tuning ξ factors in the scalar density ρ_α , since in this case their effect is very similar to that of the formation time, which switches on/off the interaction at various stages and thereby effectively changes its strength.

D. Freeze-Out

The hydrodynamic simulation is terminated by a freeze-out procedure. Though this method (as applied to high-energy physics) was first proposed almost 50 years

ago [31], this is still an actual problem which is actively discussed. The method was intuitively clear and easily applicable. However, as was shown by Cooper and Frye [32], the Milekhin's method violates the energy conservation. To remedy the situation, they proposed their own recipe. The Cooper–Frye recipe [32] was not free of problems as well. It gives negative contribution to the particle spectrum in some kinematic regions in which the normal vector to the freeze-out hyper-surface is space-like. This negative contribution corresponds to frozen out particles returning to the hydro phase. Cut off of this negative contribution again returns us to the violation of the energy conservation. To get rid of this negative spectrum, there was proposed a modification of the Cooper–Frye recipe based on a cut-Jüttner distribution [33, 34, 35, 36]. In this distribution the part of the Jüttner distribution that gave the negative spectrum is simply cut off. To preserve the particle and energy conservation, the rest of Jüttner distribution is renormalized, effectively resulting in a new temperature and chemical potential (so called "freeze-out shock"). In fact, this cut-Jüttner recipe has no physical justification, except for practical utility. Moreover, the cut-Jüttner recipe is not supported by schematic kinetic treatment [37] of the transitional region from hydro regime to that of dilute gas and looks like a violence to the nature, making the freeze-out procedure uncontrollable. Recently there was proposed a new freeze-out recipe, a cancelling-Jüttner distribution [38], which complies with results of schematic kinetic treatment [37]. However, very recently the authors reported [39] that this cancelling-Jüttner distribution is satisfactory only for the space-like freeze-out, while it fails for the time-like one. It should be stressed that this was precisely the schematic kinetic treatment. This region, where the transition from highly collisional dynamics to the collisionless one occurs, is highly difficult for the kinetic treatment and hardly allows any justified simplifications.

All the above considerations of the freeze-out process proceeded from assumption of existing some continuous hyper-surface separating the hydro system from the frozen-out gas. Conservation conditions on such hyper-surface are constructed in analogy with shock front in hydrodynamics. From the practical point of view, it would mean that we should first run the hydro calculation without any freeze-out and only after that look for a hyper-surface, where the freeze-out criterion is met. In practice our hydro simulation proceeds in different way. The freeze-out criterion is checked continuously during the simulation. If some parts of the hydro system meet this criterion, they decouple from the hydro calculation. The frozen-out matter escapes from the system, removing all the energy and momentum accumulated in this matter. Therefore, it produces no recoil to the rest of still hydrodynamic system [107]. In particular, it means that the boundary condition on the free surface (between the hydro system and vacuum) is not kept at the same position as in the calculation without freeze-out but moves inside the system. It affects not only the system surface

but also the interior. The freeze-out process looks like an evaporation (or fragmentation, on account of final-size grid) first from the system surface and then as a volume fragmentation of the system residue.

In view of above said, we prefer to avoid not quite justified complications of the freeze-out procedure and make use of the simplest (however equally unjustified) original choice of Milekhin [31] with corrected treatment of the energy conservation. The freeze-out criterion we use is

$$\varepsilon_{\text{tot}} < \varepsilon_{\text{frz}}, \quad (30)$$

where

$$\varepsilon_{\text{tot}} = \left(T_{\text{p}}^{00} + T_{\text{t}}^{00} + T_{\text{f}}^{(\text{eq})00} \right)_{\text{proper}} \quad (31)$$

is the energy density of all three fluids in the proper reference frame, where all nondiagonal components of the total energy–momentum tensor are zero,

$$\left(T_{\text{p}}^{\mu\nu} + T_{\text{t}}^{\mu\nu} + T_{\text{f}}^{(\text{eq})\mu\nu} \right)_{\text{proper}}^{\mu \neq \nu} = 0, \quad (32)$$

and ε_{frz} is the critical freeze-out energy density. If the freeze-out criterion is met in some space–time point, all three fluids in this point get frozen out and removed from the hydrodynamic evolution. In fact, we freeze out the fluids in tiny portions, i.e. droplets. This is allowed and even implied by the numerical scheme we use (see App. A). Each droplet gets frozen out in its proper reference frame. In terms of the hypersurface, our freeze-out hyper-surface is discontinuous and consists of small fragments with normal vectors of local 4-velocities. Any transport is prohibited on the non-existent parts (parallel to the flow velocity) of the surface. In this sense, our freeze-out is more similar to a continuous fragmentation of the system rather than to occurrence of the shock front.

A frozen-out droplet of the α -fluid (let it be marked as $i\alpha$) is still characterized by some temperature, baryon and strange chemical potentials corresponding to a non-gas EoS (involving some mean fields) used in the hydro calculation. This is not suitable for calculation of the spectrum of observable particles. First we should release the energy stored in mean fields. To do this, we recalculate temperature ($T^{i\alpha(\text{gas})}$), baryon ($\mu_b^{i\alpha(\text{gas})}$) and strange ($\mu_s^{i\alpha(\text{gas})}$) chemical potentials corresponding to the hadronic gas EoS proceeding from conservations of total energy–momentum, baryon and strange charges in the droplet. This has been done in the following way. From the solution of hydrodynamic equations (5)–(7) and (12) we know five quantities for each fluid: J_{α}^0 , T_{α}^{00} , T_{α}^{01} , T_{α}^{02} and T_{α}^{03} . In order to interpret them in thermodynamic terms, we should first determine six quantities for each fluid: n_{α} , ε_{α} , P_{α} and 3 components of the 4-velocity u_{α} , proceeding from above five hydrodynamic quantities, see Eqs. (10) and (11). Naturally, five equations are not enough for determining six quantities. Therefore, we add one more equation to this set—the EoS.

Thus, the resulting thermodynamic quantities turn out to be EoS dependent. In particular, this scheme with the gas EoS results in a change of the hydrodynamic 4-velocity ($u_{i\alpha(\text{gas})}^\mu$) as compared to that calculated with the EoS used in the hydrodynamic simulation. The second, already conventional step consists in determination of temperature and chemical potentials proceeding from

baryonic, strange and energy densities and pressure.

This is a kind of “freeze-out shock”, which however is completely different from that induced by the cut-Jüttner recipe. Now in terms of frozen-out droplets of various α -fluids, the spectrum of observable hadrons of a species with e_b^a baryon and e_s^a strange charges can be expressed as follows

$$E \frac{dN_a}{d^3p} = \sum_{i\alpha} \frac{g_a V_{i\alpha}^{(\text{proper})}}{(2\pi)^3} \frac{p_\mu u_{i\alpha(\text{gas})}^\mu}{\exp \left\{ \left(p_\mu u_{i\alpha(\text{gas})}^\mu - e_b^a \mu_b^{i\alpha(\text{gas})} - e_s^a \mu_s^{i\alpha(\text{gas})} \right) / T^{i\alpha(\text{gas})} \right\} \pm 1} \quad (33)$$

where g_a is degeneracy of the a particle, $V_{i\alpha}^{(\text{proper})}$ is the volume of the $i\alpha$ -droplet in its rest frame, and the sum runs over all frozen-out droplets of all fluids. If the a species is a baryon, the upper sign (+) should be taken in the denominator, if it is a meson, the lower sign (−).

The described “fragmentation” method of freeze-out precisely conserves energy–momentum and various charges. However, it is also not free from problems. The microscopic justification of this method is still lacking. The problem of returning frozen-out particles into the hydrodynamic phase still persists. Certainly, further search for a reliable freeze-out procedure is needed. A more consistent way of performing the freeze-out, the method of “continuous emission”, was proposed in Ref. [40]. Avoiding sharply defined freeze-out hypersurface, this method considers a continuous emission of particles from a finite volume, governed by their mean free paths. Its predictions at the level of observables differ from those based on the Cooper–Frye recipe [41]. Unfortunately, this method is very difficult for the numerical implementation.

III. SIMULATIONS OF NUCLEUS–NUCLEUS COLLISIONS

The strategy of our simulations is as follows. At the first step, we try to reproduce the stopping power observed in proton rapidity distributions by means of fine fitting of friction forces of the model. In principle, friction forces are EoS dependent both through scalar densities (21) and because of medium modifications of cross sections. In spite of this, our friction is just schematically estimated proceeding from vacuum proton–proton cross sections and therefore does not comply with the EoS used. This is so even for the simplest hadronic EoS, used in the present paper. The list of uncertainties relevant to the friction forces has been discussed in Subsect. IIB. In view of this, the strategy of fine fitting of friction forces is quite reasonable. Note that the success of such fit is not obvious in advance because by means of two functions $\xi_h(s)$ and $\xi_q(s)$ of a single variable (in the

present case, only $\xi_h(s)$), see Eq. (21), we fit a function of three variables: rapidity, incident energy and impact parameter. It is worthwhile to mention that the fit of the friction is EoS dependent. It allows to reproduce proton rapidity distributions only with the particular EoS. In general, another EoS requires different fit.

After the friction forces have been fixed, we tune the freeze-out energy density (in the reasonable range) in order to reproduce transverse-mass proton spectra and multiplicities of produced pions at comparatively low incident energies $E_{\text{lab}} \lesssim 20A$ GeV. At higher incident energies, the pion multiplicity turns out to be weakly sensitive to variation of the freeze-out energy density. In principle, the freeze-out criterion could be different for different particle species and even for the chemical and thermal freeze-out. However, we keep it unique for all the cases in order to avoid multiplication of fitting parameters. Therefore, fit of these quantities by means of a single parameter is not an obvious task. At higher incident energies $E_{\text{lab}} \gtrsim 30A$ GeV the pion multiplicities become sensitive to the formation time τ . Therefore, the next step consists in tuning τ (again in the reasonable range) to reproduce them already at higher incident energies. These subsequent steps are simplified by the fact that proton rapidity distributions are only slightly sensitive to variation of τ and the freeze-out energy density. After all these steps, all the model parameters got fixed, and all further calculations give pure predictions of the model. Our final aim is to find a EoS which reproduces in the best way the largest body of the observables of nuclear collisions in the incident energy range $E_{\text{lab}} \simeq (1\text{--}160)A$ GeV.

A. Hadronic EoS

We start our simulations with the purely hadronic EoS. This EoS, which was originally used in 2-fluid simulations [10, 11, 12], was proposed in [46]. It is a natural reference point for any other more elaborate EoS. The energy

density and pressure are constructed as follows:

$$\varepsilon(n_B, T) = \varepsilon_{\text{gas}}(n_B, T) + W(n_B), \quad (34)$$

$$P(n_B, T) = P_{\text{gas}}(n_B, T) + n_B \frac{dW(n_B)}{dn_B} - W(n_B), \quad (35)$$

where $\varepsilon_{\text{gas}}(n_B, T)$ and $P_{\text{gas}}(n_B, T)$ are the energy density and pressure of relativistic hadronic gas, respectively, which depend on baryon density n_B and temperature T . The only difference from the ideal gas is that baryons are affected by a mean field $U(n_B)$, i.e. the energy of the a -baryon of mass M_a with momentum \mathbf{p} is $\epsilon_a = (\mathbf{p}^2 + M_a^2)^{1/2} + b_a U(n_B)$, where b_a is the baryon number of the a -particle, and the potential $U(n_B)$ is parametrized as follows

$$U(n_B) = m_N \left[-2b \left(\frac{n_B}{n_0} \right) + c(\gamma + 2) \left(\frac{n_B}{n_0} \right)^{\gamma+1} \right]. \quad (36)$$

It depends only on the density n_B . The self-consistent potential contribution to the energy density, $W(n_B)$, is

$$W(n_B) = \int_0^{n_B} U(n) dn. \quad (37)$$

Parameters b , c and γ are determined from the condition that the cold nuclear matter saturates at $n_0 = 0.15 \text{ fm}^{-3}$ and $\varepsilon(n_0, T=0)/n_0 - m_N = -16 \text{ MeV}$, and incompressibility of this nuclear matter is $K = 210 \text{ MeV}$. Parametrization (36) results in superluminal sound ve-

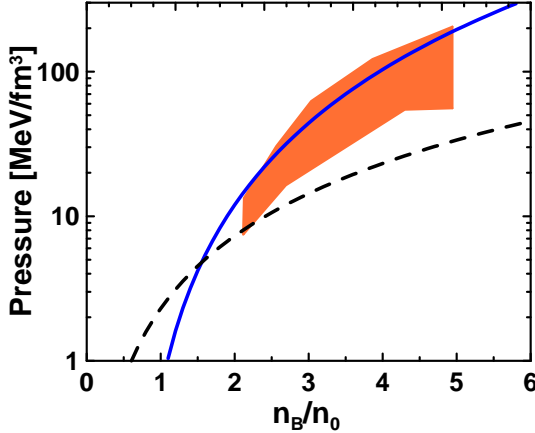


FIG. 1: (Color online) Baryon-density dependence of the pressure at $T = 0$. Shaded region is the constraint [47] derived from experimental data. Solid and dashed lines present the pressure $P(n_B, T = 0)$ and that for the ideal hadronic gas, $P_{\text{gas}}(n_B, T = 0)$ (cf. Eq. (35)), respectively.

locity at densities $n_B/n_0 > 8$. To preserve causality at high n_B , the following form of the energy density

$$\varepsilon(n_B, T=0) = n_0 m_N \left[A \left(\frac{n_B}{n_0} \right)^2 + C + B \left(\frac{n_0}{n_B} \right) \right] \quad (38)$$

is used at $n_B/n_0 > 6$. Parameters A , B and C are determined on the condition that $\varepsilon(n_B, T=0)$ and its two

first derivatives are continuous at $n_B/n_0 = 6$. As seen from Fig. 1, the pressure of this hadronic EoS is within the constraint given by Danielewicz et al. extracted from the analysis of flow of nuclear matter [47] at the AGS incident energies.

B. Summary of Parameters: Friction, Freeze-Out, etc.

For the sake of complete account, in this section we would like to summarize the parameters used in the present simulations. The specific reasons for choosing these parameters will be discussed in subsequent Subsections.

- The key quantity is the EoS, which was taken in the simple form of purely hadronic EoS, see Sect. III A. In this EoS, 48 different hadronic species are taken into account. Each hadronic species includes all the relevant isospin states, e.g., the nucleon species includes proton and neutron.
- The friction between baryon-rich fluids was fitted to reproduce the stopping power observed in proton rapidity distributions, see Sect. III C. For this purpose the original friction, parametrized through experimental inclusive proton-proton cross sections [29], was enhanced by means of the tuning factor ξ_h

$$\xi_h^2(s) = \gamma_h + 2\beta_h \left[\ln(s/(2m_N)^2)^{1/2} \right]^{1/4} \quad (39)$$

with $\gamma_h = 1$ and $\beta_h = 0.75$, see Fig. 2. The ξ_q factor is not applicable here because of the pure hadronic nature of the EoS.

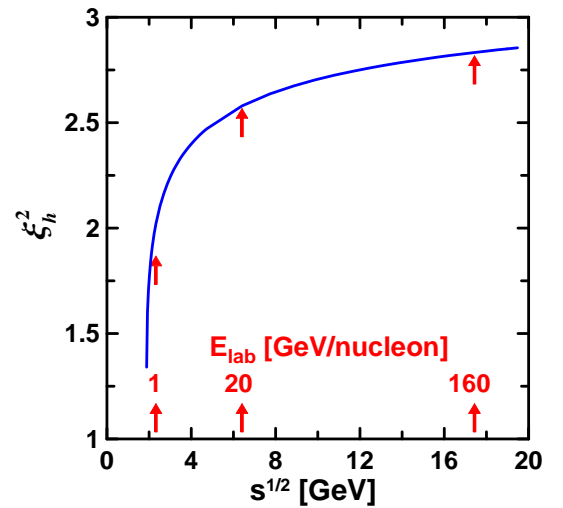


FIG. 2: (Color online) Fitted friction enhancement for the purely hadronic EoS [46] used in simulations, see Eq. (21).

- The parameter of freeze-out energy density was fitted to reproduce transverse-mass proton spectra, see Subsect. IIID, and multiplicities of produced pions at comparatively low incident energies $E_{\text{lab}} \lesssim 20A$ GeV, see Subsect. IIIE. It has been taken $\varepsilon_{\text{frz}} \simeq 0.2$ GeV/fm³ at all incident energies with the only exception: $\varepsilon_{\text{frz}} \simeq 0.1$ GeV/fm³ at $E_{\text{lab}} \leq 2A$ GeV. These ε_{frz} values are presented as approximate quantities because of the numerical realization of the freeze-out procedure (cf. App. A).
- The formation time of the fireball fluid was also fitted to reproduce the pion yield at higher incident energies $E_{\text{lab}} \gtrsim 30A$ GeV, see Subsect. IIIE. It has been taken $\tau = 2$ fm/c. Note that τ is the formation time of a fluid element, consisting of a number of particles, which are generally not thermalized. This implies that the formation time of a separate particle is certainly shorter than τ .
- The contributions of strong decays of hadronic resonances (R) into spectra of stable hadrons is taken into account, as it is described in App. B. In the present calculation all spectral functions were taken without width

$$A_R(s) = \delta(s - m_R^2), \quad (40)$$

where m_R is the mass of the R resonance.

- For the reproduction of stable hadron multiplicities it is also important to take into account contributions of weak decays with small values of $c\tau_w$, where τ_w is the inverse width of the decay. Feed back from weak decays of hadrons with $c\tau_w < \text{few cm}$, i.e. K_{short}^0 , Λ , Σ , $\bar{\Lambda}$ and $\bar{\Sigma}$ [108] have been taken into account in yields of stable particles. In particular, this is important for reproduction of the pion multiplicities (Subsects IIIE and IIH) and the proper normalization of proton spectra (Subsects IIIC and IIID). On the other hand, weakly decaying hadrons with $c\tau_w$ of the order of few meters, i.e. K_{long}^0 , K^+ and K^- , were treated as stable particles.
- Light fragment formation (deutrons, tritons, ^3He and ^4He) is taken into account in terms of the coalescence model, which is similar to that in Appendix E of Ref. [16]. In Ref. [16], the coalescence coefficients were fitted at the incident energy 0.8A GeV. To accommodate this formulation to different incident energies, we change only overall scale of the coalescence (the coefficient C_{coal}) which either enhances ($C_{\text{coal}} > 1$) or reduces ($C_{\text{coal}} < 1$) its strength, keeping ratios of yields of various fragments the same as in [16], see Table I. At $E_{\text{lab}} > 9A$ GeV we do not apply any coalescence to calculations of nucleon observables, since the respective correction is negligible.

E_{lab}, A GeV	1	2	4	6	8	> 9
C_{coal}	1.5	1.2	0.8	0.4	0.1	0

TABLE I: Coalescence parameters C_{coal} used for simulations of Au+Au and Pb+Pb collisions at various incident energies E_{lab} .

In order to compare with available experimental data, impact parameters used in calculations were taken either from experimental works, if they were evaluated there, or estimated based on the experimentally declared percentage of the total reaction cross section, corresponding to a particular event selection.

C. Proton Rapidity Distributions: Observable Stopping Power

The nucleon rapidity distribution basically reflects the stopping power achieved in a nuclear collision. It is defined as

$$\frac{dN_N}{dy} = \int d^2p_T \left[E \frac{dN_N}{d^3p} + \sum_R E \frac{d^3N_N^{(R \rightarrow N+X)}}{d^3p} \right] \quad (41)$$

in terms of frozen-out spectra of nucleons EdN_N/d^3p , see Eq. (33), and baryonic resonances. The second term under the integral takes into account the contribution of these resonance decays, cf. Eq. (B5), App. B, the sum runs over all baryonic resonances R . Integration runs over the transverse (with respect to the beam) momentum p_T .

Either identified protons or difference between positive and negative hadrons are experimentally measured. The latter is associated with the proton-antiproton difference, $(p - \bar{p})$. Since particles are not isotopically distinguished in our model, we estimate the proton distribution simply as

$$\frac{dN_p}{dy} = \frac{Z}{A} \frac{dN_N}{dy}, \quad (42)$$

where Z and A are the proton and mass numbers, respectively, in the colliding (identical) nuclei. This estimate is quite reasonable at comparatively low incident energies. At higher energies, when abundant particle production starts, this recipe somewhat underestimates the proton number. The reason is that newly produced particles tend to restore the isotopic symmetry in the system and hence increase the number of protons as compared with its initial value. The $(p - \bar{p})$ quantity is much less sensitive to the effect of newly produced particles, since their contribution is essentially canceled in the difference $(p - \bar{p})$. Therefore, the Z/A scaling of the nucleon-antinucleon difference is a reasonable approximation for the $(p - \bar{p})$ quantity even at high incident energies. As mentioned above, our aim is to reproduce these distributions in a

wide range of incident energies from 1A to about 160A GeV.

In general, nucleon observables are more robust to variations of physical parameters than other probes, since they are essentially confined by the baryon number conservation. Our first observation is that the nucleon and, in particular, proton rapidity distributions are only weakly sensitive to variation of the freeze-out energy density ε_{frz} and the formation time of the fireball fluid τ . As for ε_{frz} , there is a certain compensation between effects of collective motion and internal excitation. Both these effects produce similar consequences in nucleon spectra. For instance, if we allow the system to evolve longer (the

lower ε_{frz}), the collective motion of the matter becomes more developed while its internal excitation drops down (i.e. the matter cools down). This counteraction results only in slight changes in nucleon rapidity distributions. As for the formation time, the density of the produced fireball fluid is not high enough even at the highest considered energy. Therefore, its interaction with baryon-rich fluids only slightly affects the baryon subsystem [23]. This weak dependence on ε_{frz} and τ allows us to use nucleon rapidity distributions to fit the observable stopping power, to which nucleon observables are indeed sensitive.

The second main conclusion is that the original friction between baryon-rich fluids, estimated in Ref. [29] proceeding from free proton-proton cross sections, is evidently insufficient to reproduce observable stopping of nuclear matter. Therefore, this friction is fitted to reproduce this stopping power observed in proton rapidity distributions. For this purpose the original friction was enhanced by means of the tuning factor ξ_h , cf. Eq. (39). Thus, the friction enhancement is the larger, the higher incident energy is. At the lowest considered energy of 1A GeV, the friction turns out to be approximately 2 times enhanced. This is similar to earlier results of the 2-fluid model with mean mesonic fields [16]. There it was found out that for the proper reproduction of data on heavy-ion collisions in the energy range from 0.4 to 0.8A GeV the enhancement factor of 3 was required.

Fig. 3 illustrates the overall quality of reproduction of experimental data by our hydrodynamic calculations as well as by other transport simulations for two basic incident energies of 10 and 160A GeV. The 3-fluid simulations were performed at impact parameters which correspond to the experimentally declared fraction of the total reaction cross section, accumulated in the experimentally selected central events, assuming sharp cut off in impact parameters. The side bumps of the HSD calculations correspond to spectator parts of colliding nuclei. In our model, these spectator parts were cut off, based on simple criterion: the energy per baryon is less than the nucleon mass, which is the case, when a nucleon is at least loosely bound in the matter. As seen, experimental data from different experiments somewhat differ, and therefore one should not expect their reproduction in the model calculations within better than $\sim 10\%$. Uncertainties in the impact parameter at the level of about 0.5 fm do not noticeably affect hydrodynamic rapidity spectra. In general, there is a reasonable agreement between hydrodynamic and kinetic calculations.

Comparison with experimental data for identified protons is presented in Fig. 4 for the SIS-AGS energy range, where various selections of non-central nuclear collisions are also considered. Here and below, impact parameters for each set of non-central interactions were chosen accordingly the experimentally declared fractions of the total reaction cross section, accumulated in the set of experimentally selected events, assuming sharp cut off in impact parameters.

At the beam energies E_{lab} of the order of 1A GeV, a

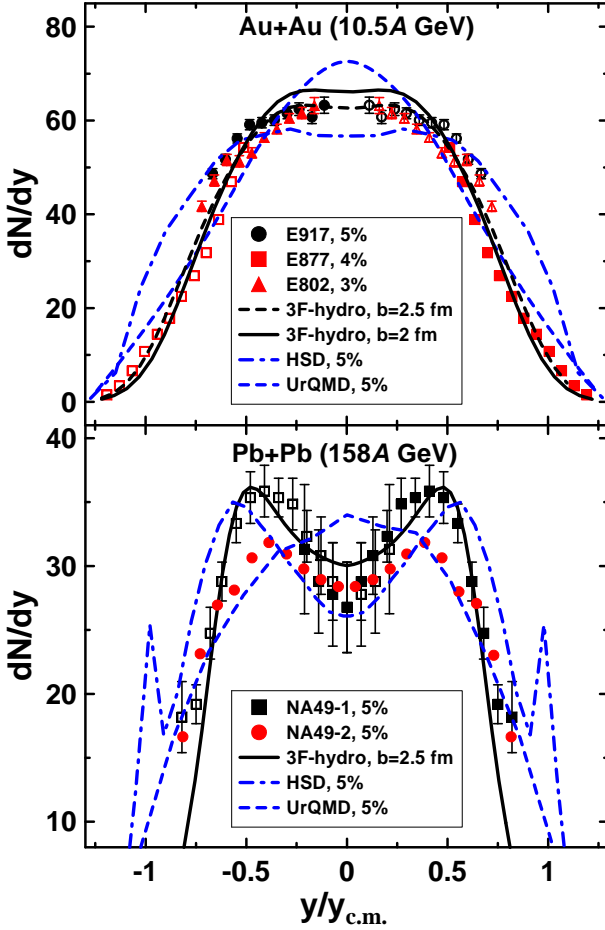


FIG. 3: (Color online) Rapidity spectra of protons (upper panel) and $(p - \bar{p})$ (lower panel) from central heavy-ion collisions. Bold solid lines and bold dashed line (upper panel) correspond to 3-fluid hydrodynamic calculations at different impact parameters. Thin dashed-dotted and short-dashed lines are the appropriate UrQMD and HSD model results [48], respectively. Full symbols display measured experimental points, whereas the open ones are those reflected with respect to the mid rapidity point. Experimental data are taken by collaborations E802 [49], E877 [50], E917 [51], NA49 [52, 53, 54]. The percentage shows the fraction of the total reaction cross section, corresponding to experimental selection of central events.

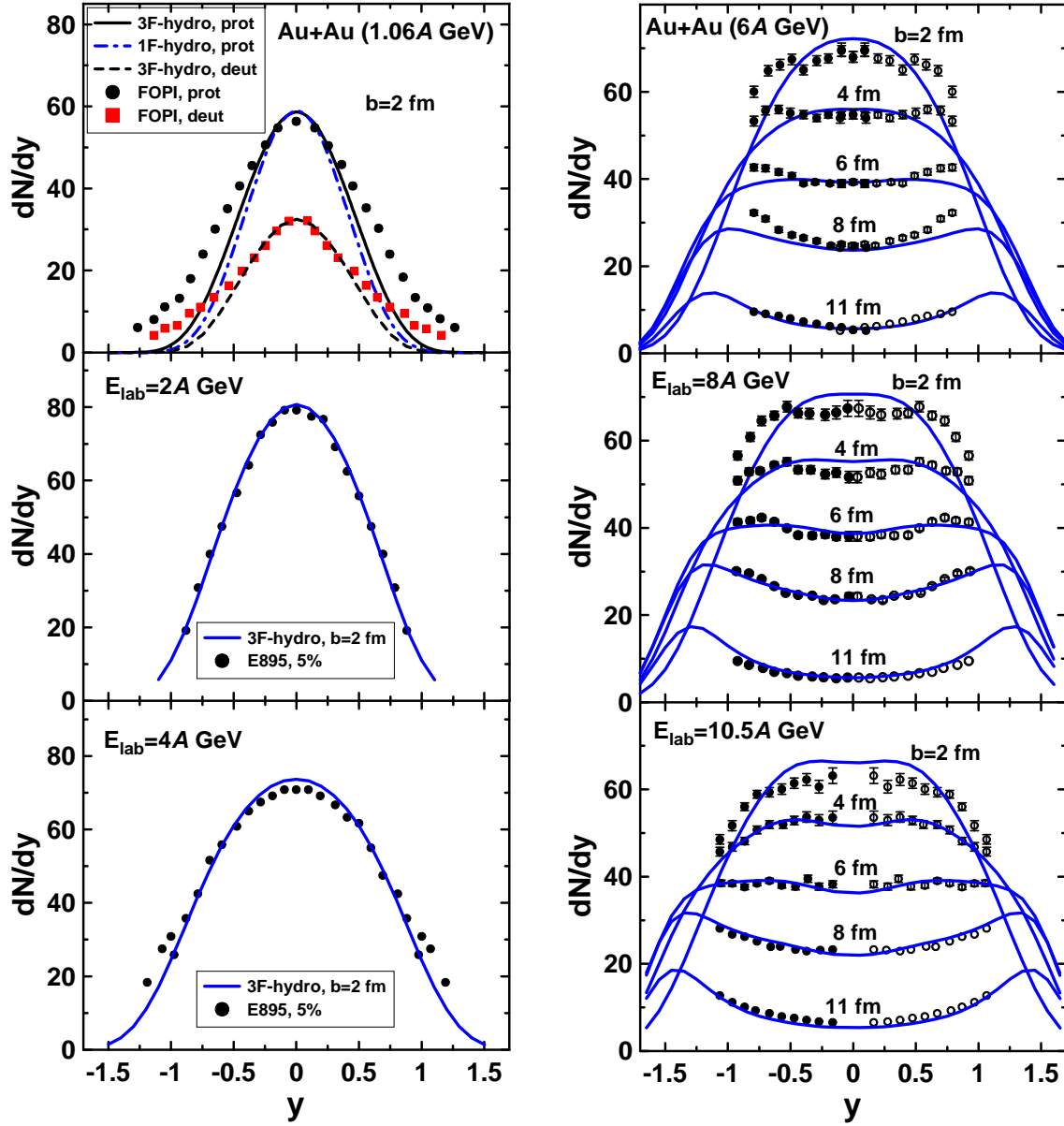


FIG. 4: (Color online) Proton rapidity spectra at SIS and AGS energies for various impact parameter. Solid lines correspond to 3-fluid calculations. The dashed line is that for deuteron production. For comparison the 1-fluid result is shown by the dotted-dashed line at $E_{\text{lab}} = 1.06A$ GeV. Experimental points at different energies are taken from [55] at $E_{\text{lab}} = 1.06A$ GeV, [56] at 2A and 4A GeV, and [51] at 6A, 8A and 10.5A GeV. The percentage indicates the fraction of the total reaction cross section, corresponding to experimentally selected events.

large fraction of nucleons is produced with low relative velocities. Therefore, they may coalesce forming light fragments. The coalescence formulation accepted here is similar to that in Appendix E of Ref. [16]. Only light fragments (deutrons, tritons, ^3He and ^4He) are taken into account. To fit this formulation to different incident energies, we change only overall scale of the coalescence (the coefficient C_{coal}) which either enhances ($C_{\text{coal}} > 1$) or reduces ($C_{\text{coal}} < 1$) its strength, keeping ratios of yields of various fragments the same as in [16], see Table I.

At the SIS energy we describe reasonably well both

proton and deuteron rapidity spectra. The fact that the experimental distribution is somewhat wider than the calculated one can be explained by that the set of experimentally selected events in fact contains a certain admixture of semi-central and peripheral events, whereas the calculation was performed for the single central impact parameter. Indeed, at comparatively low energies the reliable selection of central events is problematic [16]. This non-central admixture makes the distributions wider as compared to what it would be for the perfect central selection. At this energy the stopping power is rather high.

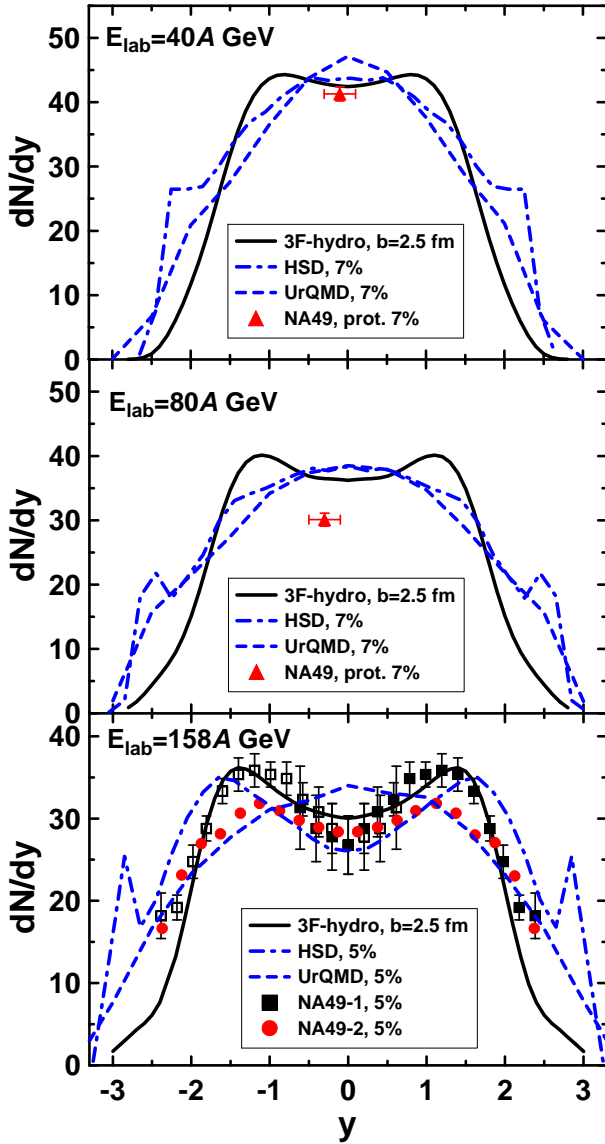


FIG. 5: (Color online) $(p - \bar{p})$ rapidity distribution from central Pb+Pb collisions at SPS energies. Solid lines are 3-fluid calculations, whereas dashed-dotted and dashed lines are the corresponding predictions of the HSD and UrQMD models [48], respectively. NA49 experimental data are from Ref. [54] for $E_{\text{lab}} = 40A$ and $80A$ GeV and from Refs. [52, 53, 54] for $E_{\text{lab}} = 158A$ GeV. The percentage indicates the fraction of the total reaction cross section, corresponding to experimentally selected events.

This is seen from both the Gaussian-like shape of the rapidity distributions and also from the fact that the 3-fluid results are quite close to those of the conventional 1-fluid calculations (see Fig. 4). A separate code was used for these 1-fluid calculations. With the increase of the incident energy, the spectrum shape starts to differ from the Gaussian one, getting more and more flat at the mid rapidity. The 3-fluid model reasonably reproduces the dependence of the spectra on both the incident energy E_{lab}

and the impact parameter b . Note that no extra tuning of normalization of the spectra was done. The values of impact parameters were taken from the experimental estimate of centrality of nuclear interactions [51]. Though there are some uncertainties in this estimate, we applied no special tuning to values of these impact parameters.

The $(p - \bar{p})$ rapidity spectra at the SPS energies are shown in Fig. 5. A minimum of hydrodynamic distributions dN/dy in the mid rapidity region complies with experimental observations at $E_{\text{lab}} = 158A$ GeV. It survives at lower incident energies, up to $E_{\text{lab}} = 40A$ GeV. This is in contrast to simulations based on kinetic transport codes [48], which predict flat or even peaked distributions in the mid rapidity region at $E_{\text{lab}} = 40A$ and $80A$ GeV. Available experimental points here do not allow us to verify these different predictions. Note that experimental points at $E_{\text{lab}} = 40A$ and $80A$ GeV correspond to identified protons, while all presented calculations are related to $(p - \bar{p})$.

D. Proton Transverse Mass Distributions

Once the friction has been already fitted to reproduce proton rapidity distributions, we can vary only the freeze-out energy density ε_{frz} and the formation time of the fireball fluid τ . In contrast to rapidity distributions, the proton transverse-mass distributions turn out to be more sensitive to ε_{frz} , since their slopes reflect the effective temperature of the frozen-out distributions. This is because the internal excitation certainly dominates in the transverse direction as compared to the collective motion. At the same time, these distributions are rather insensitive to τ , as well as all other baryonic (however, not antibaryonic) quantities. Therefore, to reproduce them, we choose $\varepsilon_{\text{frz}} \simeq 0.2$ GeV/fm³ at all incident energies with the only exception: $\varepsilon_{\text{frz}} \simeq 0.1$ GeV/fm³ at $E_{\text{lab}} \leq 2A$ GeV. As for low energies, $E_{\text{lab}} \leq 2A$ GeV, we had to reduce ε_{frz} , since otherwise the freeze-out would occur at the very early stage of the collision. The approximate nature of the ε_{frz} values results from the numerical realization of the freeze-out procedure (cf. App. A).

Proton transverse-mass spectra at the AGS energies are exemplified in Fig. 6. They exhibit a typical exponential fall-off. As seen, the 3-fluid model well reproduces both the normalization and the slopes of this fall-off, as well as their rapidity dependence. The right panel of Fig. 6 shows the dependence of the transverse mass distributions at the mid rapidity on the impact parameter b . As before, the values of impact parameters were taken from the experimental estimate of centrality of nuclear interaction [51]. The impact-parameter dependence is reasonably reproduced by the model as well even at relatively large impact parameters.

The beam-energy dependence of the proton transverse-mass distributions is presented in Fig. 7 for central Pb+Pb collisions at the SPS energies. Overall, these distributions are in agreement with experimental data.

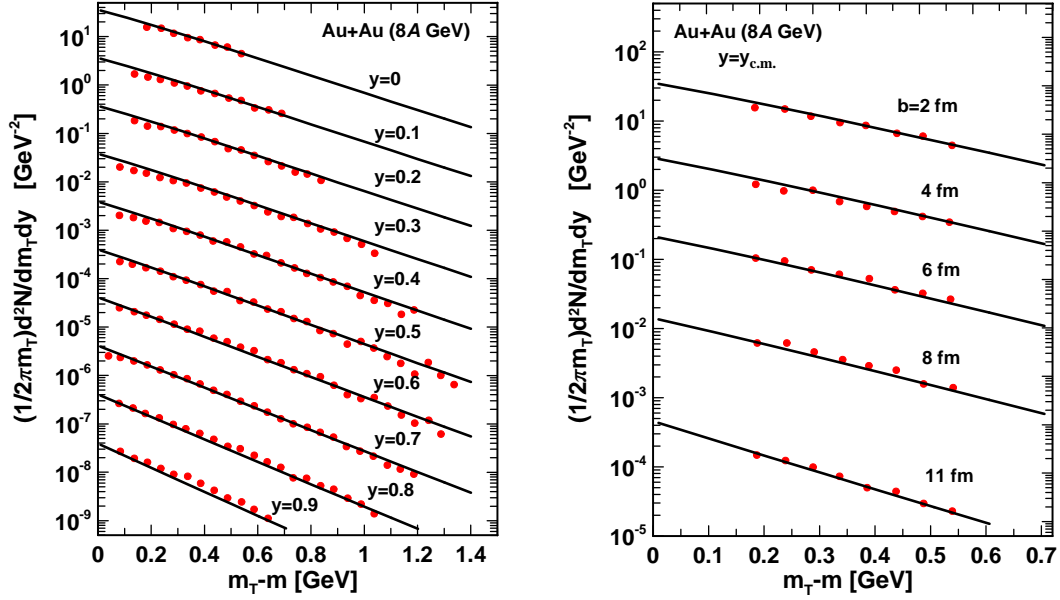


FIG. 6: (Color online) Transverse mass distributions of protons from Au(8A GeV)+Au collisions at $b = 2$ fm and different rapidities in the c.m. system (left panel) and at the mid rapidity and different impact parameters (right panel), cf. the right panel of Fig. 4. For clarity of representation, every next data set and the corresponding curve (from top to bottom) is multiplied by the additional factor 0.1. Experimental data are from [51].

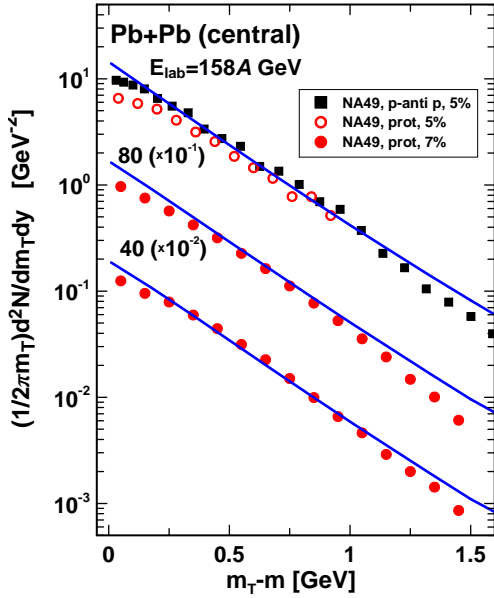


FIG. 7: (Color online) Transverse mass distributions of protons at the mid rapidity from central Pb+Pb collisions at incident energies $E_{\text{lab}} = 158A, 80A$ and $40A$ GeV and impact parameter $b = 2.5$ fm, cf. Fig. 5. NA49 experimental data are taken from [52] (squares), [57] (open circles) and [57] (full circles).

However, the 3-fluid model does not exhibit deviation from the exponential fall-off at $(m_T - m) \lesssim 0.2$ GeV, observed in experiment, most clearly at $E_{\text{lab}} = 158A$ GeV. The same feature of the hydrodynamic calculation was

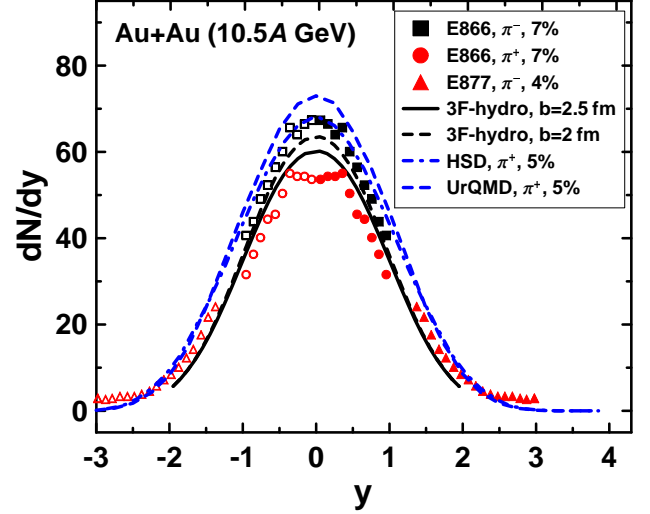


FIG. 8: (Color online) Pion rapidity spectra from central Au(10.5A GeV)+Au collisions. Bold solid and dashed lines represent 3-fluid calculations at $b = 2.5$ and 2.0 fm, respectively. Thin dashed-dotted and short-dashed lines correspond π^+ spectra from kinetic simulations within HSD and UrQMD models [48]. The π^- (full squares and triangles) and π^+ (full circles) data are measured by E895 [56] and E877 [50] collaborations. The percentage indicates the fraction of the total reaction cross section, corresponding to experimentally selected events. Open symbols are obtained by reflecting the full ones with respect to the mid rapidity.

earlier reported in Ref. [22]. As it was shown [22], a post-hydro kinetic evolution is required to produce the observable two-slope structure of the m_T -spectra. In our model such a post-hydro evolution is absent. It is

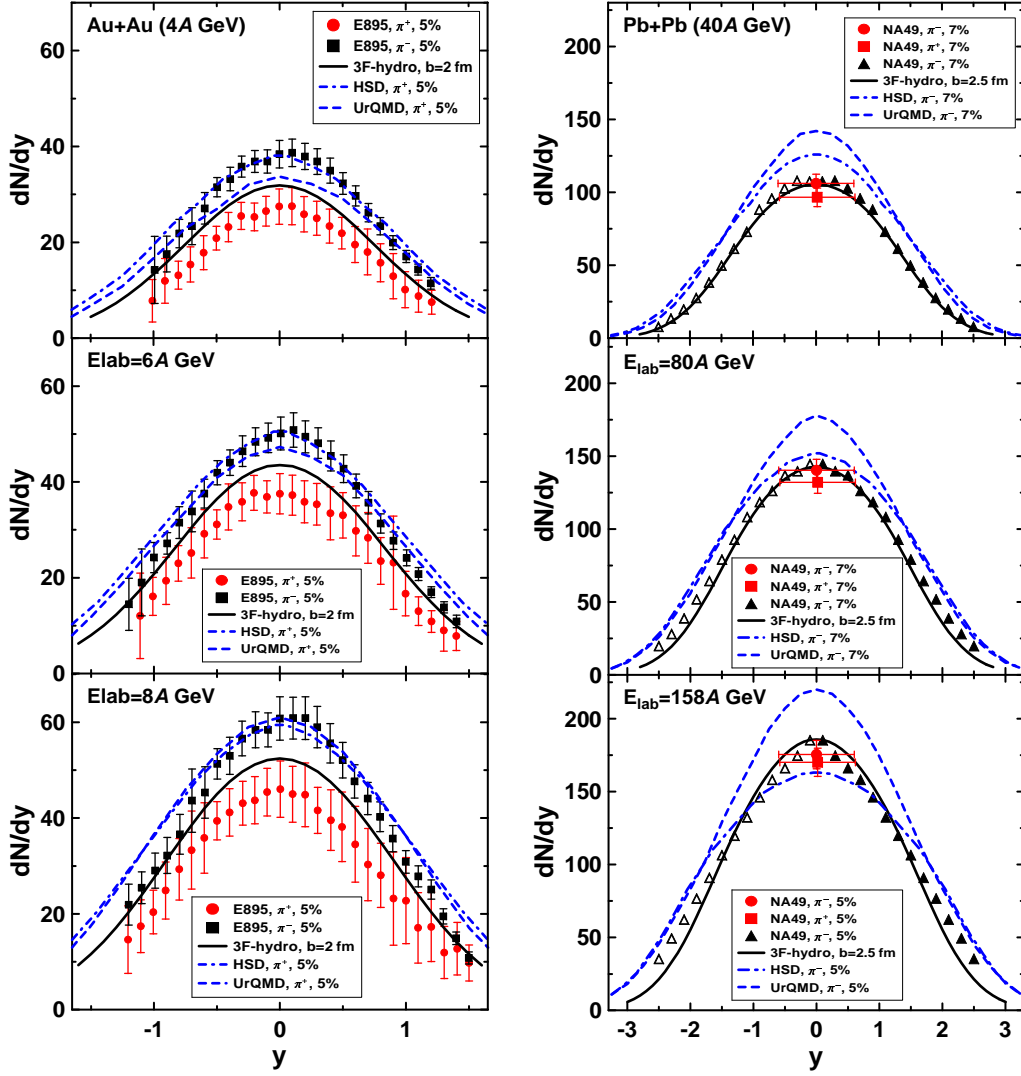


FIG. 9: (Color online) Pion rapidity spectra from central collisions at the AGS (left panel) and SPS (right panel) energies. Experimental points are taken from [56] (AGS energies) and [58] (SPS energies). Notation is the same as in Fig. 8.

worthwhile to note that the above problem is not an inalienable feature of any hydrodynamic calculation. For instance, the deviation from the exponential fall-off was reproduced in calculations by Kolb *et al.* [21].

E. Pion Rapidity Distributions

Production of new particles is closely related to the amount of entropy accumulated in the system. At the late stage of the collision, the system expansion is isentropic, i.e. the total entropy is conserved. This is indeed so in our simulations, as we have checked it. This fact implies that at high incident energies $E_{\text{lab}} \gtrsim 40A$ GeV the total number of produced pions is approximately conserved during this expansion stage, since thermal pions are the dominant component among the produced particles. From the practical point of view, it means that the

pion number is approximately independent of the freeze-out energy density ε_{frz} . This is indeed observed in actual simulations. However, at high incident energies the pion number depends on the formation time of fireball fluid τ , since 20÷30% of the pions are produced in the fireball fluid. The shorter τ is, the earlier the fireball fluid starts to interact with baryonic subsystem, and hence the fewer pions survive in this fluid. This implies that the formation time effectively tunes the strength of the fireball-projectile(target) friction which was only roughly estimated (cf. subsect. II C).

At the first glance, the above speculation contradicts to the pion absorption mechanism (cf. in Subsect. II C), which assumes that pions are simply captured by the baryon-rich fluids without losing their number. However, the pion number is not a conserved quantity. In the baryon-rich fluids the energy of the captured pions is thermally redistributed (accordingly to thermal chemical

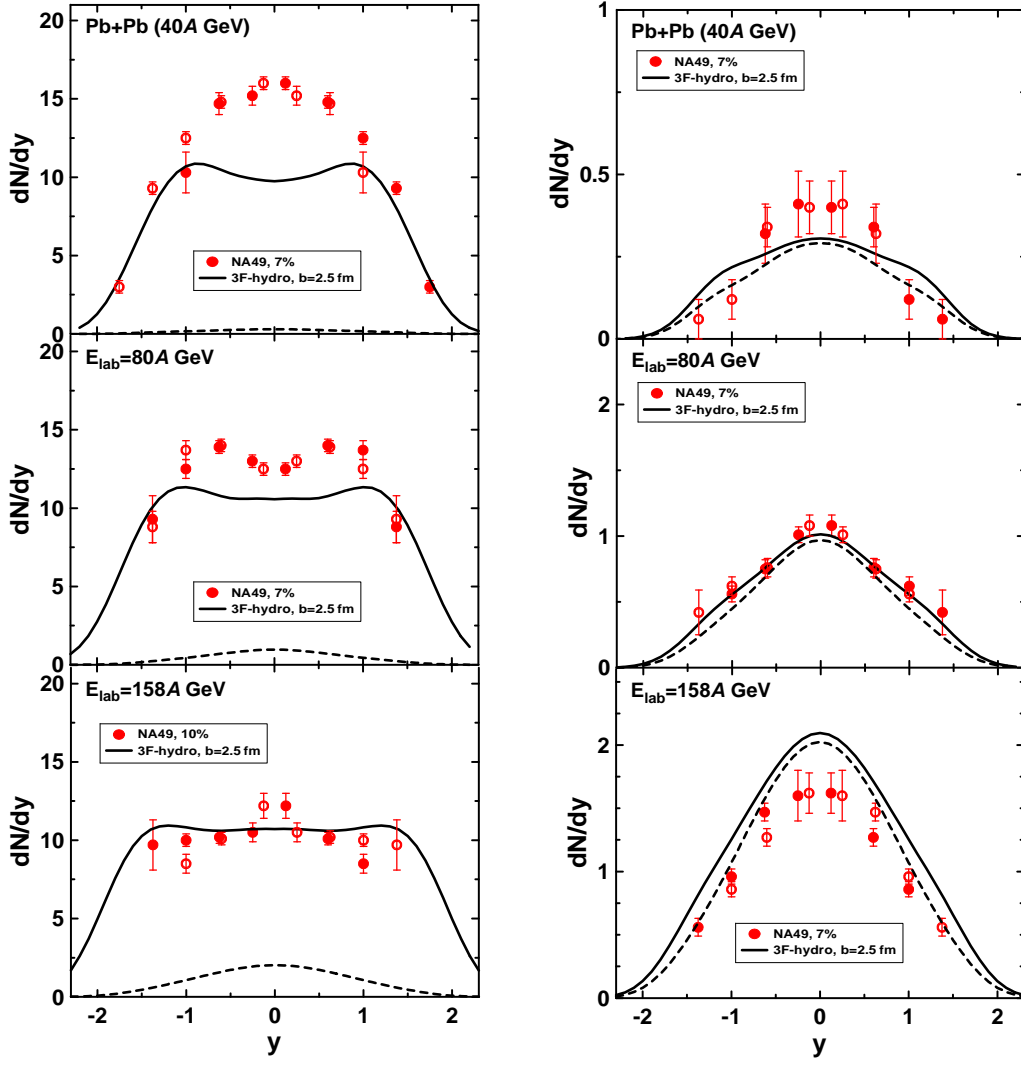


FIG. 10: (Color online) Rapidity spectra of $\Lambda + \Sigma^0$ hyperons (left panel) and $\bar{\Lambda} + \bar{\Sigma}^0$ antihyperons (right panel) from central ($b = 2.5$ fm) Pb+Pb collisions. Solid lines represent results of the 3-fluid model. Contributions from the fireball fluid are shown by dashed lines. Preliminary experimental data are taken from Ref. [59]. The percentage indicates the fraction of the total reaction cross section, corresponding to experimentally selected events.

equilibrium) between kinetic energy of (mainly) baryons and thermally produced (mainly) pions. Therefore, their number is effectively reduced, since in the baryon-free fireball fluid the same energy was mainly accumulated in thermal pions.

At lower incident energies $E_{\text{lab}} \lesssim 20A$ GeV, the thermal pion production is not already so dominant but pions are rather produced through decays of resonances. Therefore, their number starts to depend on ε_{frz} . The later freeze-out occurs, the fewer pions are produced. At the same time, the contribution of the produced fireball fluid becomes less important, and hence the τ -dependence of the pion multiplicity becomes weak. At intermediate energies $20 \lesssim E_{\text{lab}} \lesssim 40A$ GeV, there is a moderate dependence of the pion multiplicity on both ε_{frz} and τ . Note that these parameters, ε_{frz} and τ , mostly affect the overall normalization of pion rapidity spectra,

whereas the shape of these spectra mainly depend on the stopping power, which is kept fixed here.

As both the friction and ε_{frz} have been already fixed above, the pion rapidity distributions at low incident energies $E_{\text{lab}} \lesssim 20A$ GeV can be considered as predictions of the model. In order to reproduce the pion yield at high incident energies $E_{\text{lab}} \gtrsim 30A$ GeV, the formation time of the fireball fluid has been taken $\tau = 2$ fm/c. Note that τ is the formation time of a fluid element, consisting of a number of particles (cf. Eq. (14) and discussion above it). This implies that the formation time of a separate particle (pion) at rest is certainly shorter than τ . Assuming that the temperature scale of the fireball fluid is approximately 100 MeV, we estimate that the formation time of a separate pion at rest approximately equals 1 fm/c. This is in good agreement with the estimate of the string-formation time. However, one should keep in

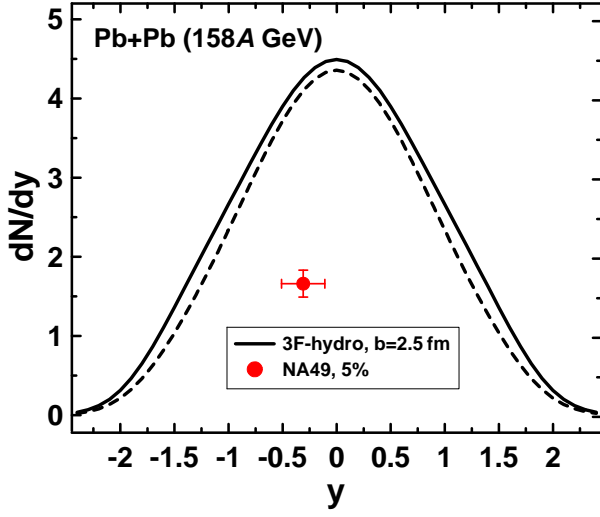


FIG. 11: (Color online) Antiproton rapidity spectrum from central Pb(158A GeV)+Pb collisions. Notation is the same as in Fig. 10. Experimental data are taken from Ref. [60].

mind that the latter value refers to a separate string in vacuum, i.e. without any effects of string interactions (junctions and color-rope formation). At the same time, our τ has an in-medium sense with all these multi-particle effects included. Therefore, their coincidence should be taken with caution. Note that with $\tau = 0$ the multiplicity of pions (as well as other newly produced particles) would be 10–15% underestimated at high incident energies ($> 30A$ GeV). All other observables would remain approximately the same.

Since particles are not isotopically distinguished in our model, we assume that numbers of pions of each charge are equal, i.e. $N_{\pi^+} = N_{\pi^-} = N_{\pi^0} = N_{\pi}/3$, where N_{π} is the calculated total number of pions. Of course, this is a rough estimate of π^+ and π^- yields, since N_{π^-} always exceeds N_{π^+} because of the initial isotopic asymmetry of colliding nuclei. Therefore, only if our calculation of $N_{\pi}/3$ complies with experimental $(N_{\pi^+} + N_{\pi^-})/2$, we refer to this as “a good agreement”.

Fig. 8 represents pion rapidity distributions in central Au+Au collisions at the incident energy $E_{\text{lab}} = 10.5A$ GeV. Sensitivity of this distribution to the variation of the impact parameter is also demonstrated. As seen, the hydrodynamic results indeed fall in between the π^+ and π^- data, hence they comply with these data. In contrast, the π^+ rapidity spectra, calculated in two transport models, HSD and UrQMD models [48] also displayed in Fig. 8, closely follow experimental points but for negative pions. The overestimate of the pion yield by the kinetic models can be attributed to a lack of collective interaction in the EoS’s corresponding to those models.

All above said is in fact true for the whole energy range under consideration, as it is demonstrated in Fig. 9. The 3-fluid model reasonably reproduces the pion distributions, while the HSD and UrQMD models certainly overestimate them.

F. Rare Particle Production

Now all the parameters of the model are fixed. Therefore, all further calculations can be treated as predictions of the 3-fluid model.

As an example of the hydrodynamic description of rare channels, the rapidity spectra of Λ and $\bar{\Lambda}$ hyperons are presented in Fig. 10. Contributions from decays of higher resonances are taken into account. Both shape and absolute value of hyperon and antihyperon spectra are reproduced surprisingly well. The Λ hyperons originate mainly from baryon-rich fluids and shapes of their spectra closely follow those of the protons. Although the experimental data are preliminary, one may note that agreement with experiment becomes certainly worse at the incident energy $E_{\text{lab}} = 40A$ GeV. Unfortunately, the only measured point in proton rapidity distribution (see Fig. 5) does not allow us to conclude on either similarity or difference of the Λ and p distributions at this energy. In contrast, $\bar{\Lambda}$ antihyperons are dominantly created in the baryon-free fireball and have typical single-bump thermal spectra.

At the same time the experimental data on antiproton production are certainly overestimated in the our model, as illustrated in Fig. 11. Since particles are not isotopically distinguished in our model, we have estimated the antiproton yield as half of that of antinucleon: $N_{\bar{p}} = \frac{1}{2}N_{\bar{N}}$, assuming that $p\bar{p}$ and $n\bar{n}$ pairs are produced with approximately equal probability. The contributions of weak-decay channels $\bar{\Lambda} \rightarrow \bar{N} + \pi$ and $\bar{\Sigma} \rightarrow \bar{N} + \pi$ are excluded.

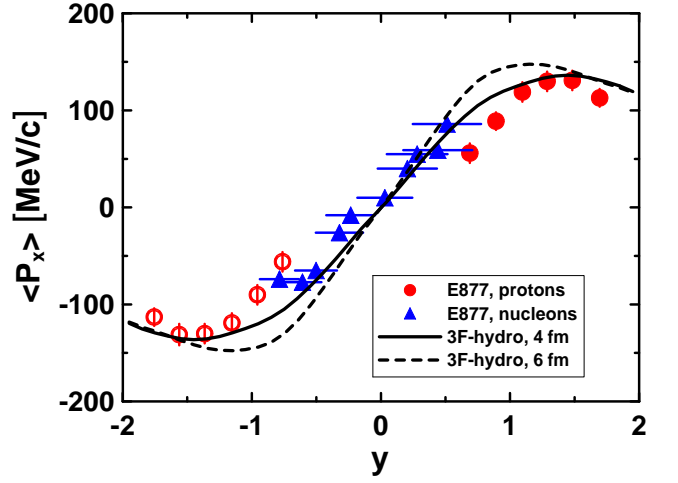


FIG. 12: (Color online) Transverse-momentum flow of nucleons and protons as a function of rapidity for semi-central Au+Au collisions at $E_{\text{lab}} = 10.5A$ GeV. The 3-fluid calculations are presented for $b = 4$ fm (solid line) and $b = 6$ fm (dashed line). The data for protons (circles) and all baryons (triangles) are taken from Ref. [63]. Full symbols correspond to measured data, open ones are those reflected with respect to the mid rapidity point.

G. Flow

Flow quantities of different types quantify space-momentum correlations of collective motion of the strongly interacting matter. This collective motion is essentially caused by the pressure gradients arising during the evolution of the collision and hence is intimately related to the EoS and, in particular, to a possible phase transitions. A spectacular loss of correlation between the

observed particle transverse momenta and the reaction plane, which gives rise to dramatic reduction of the directed flow, has been predicted by Rischke et al. [61] in the conventional hydrodynamic model with the bag-model EoS. The subsequent studies showed that the observed signal essentially depends not only on the EoS but also on the collision dynamics [12, 15].

The conventional transverse-momentum flow of an a species is defined as [62], cf. Eq. (41),

$$\langle p_x^{(a)} \rangle(y) = \frac{\int d^2 p_T p_x \left[E dN_a/d^3 p + \sum_R E d^3 N_a^{(R \rightarrow a+X)}/d^3 p \right]}{\int d^2 p_T \left[E dN_a/d^3 p + \sum_R E d^3 N_a^{(R \rightarrow a+X)}/d^3 p \right]}, \quad (43)$$

where p_x is the transverse momentum of a particle in the reaction plane, and integration runs over the transverse momentum p_T . The second term in square brackets takes into account the contribution of resonance decays, resulting in a production, cf. App. B, the sum runs over all relevant resonances R .

As seen in Fig. 12, the 3-fluid model reasonably reproduces a general trend of the baryonic $\langle p_x \rangle(y)$ distribution at $E_{\text{lab}} = 10.5A$ GeV, exhibiting two clear peaks near the target and projectile rapidities. In fact, we compute $\langle p_x \rangle(y)$ of so-called primordial nucleons, which later may

coalesce, forming light fragments. In view of this, it is not surprising that agreement with the nucleon $\langle p_x \rangle(y)$ data, which indeed take into account contribution of light nuclear fragments, seems to be better than that with identified-proton data.

At high energies the azimuthal asymmetry is usually characterized by the first and second coefficients of Fourier expansion of the azimuthal-angle dependence of the single-particle distribution function, i.e. by the directed flow $v_1 = \langle \cos \phi \rangle$ and elliptic flow $v_2 = \langle \cos 2\phi \rangle$:

$$v_1^{(a)}(y) = \frac{\int d^2 p_T (p_x/p_T) \left[E dN_a/d^3 p + \sum_R E d^3 N_a^{(R \rightarrow a+X)}/d^3 p \right]}{\int d^2 p_T \left[E dN_a/d^3 p + \sum_R E d^3 N_a^{(R \rightarrow a+X)}/d^3 p \right]}, \quad (44)$$

$$v_2^{(a)}(y) = \frac{\int d^2 p_T [(p_x^2 - p_y^2)/p_T^2] \left[E dN_a/d^3 p + \sum_R E d^3 N_a^{(R \rightarrow a+X)}/d^3 p \right]}{\int d^2 p_T \left[E dN_a/d^3 p + \sum_R E d^3 N_a^{(R \rightarrow a+X)}/d^3 p \right]}. \quad (45)$$

Examples of the v_1 and v_2 flow for protons and pions at $E_{\text{lab}} = 40A$ and $158A$ GeV are presented in Figs 13 and 14. The declared experimental percentage of the total reaction cross section, related to the experimental event selection, allows us to estimate the corresponding impact parameter as $b = 5.6$ fm. In view of uncertainty of this estimate and in order to reveal the model dependence on the impact parameter, we also show calculations with $b =$

4 fm. Flows of the fireball fluid calculated at $b = 4$ and 5.6 fm are hardly distinguishable. Several sets of data, which noticeably differ from each other, are shown in these figures. The "standard" method [62, 66] for evaluating the flow coefficients requires an event-by-event estimate of the reaction plane with which outgoing hadrons correlate. However, this method does not discriminate other sources of correlations, like those due to global mo-

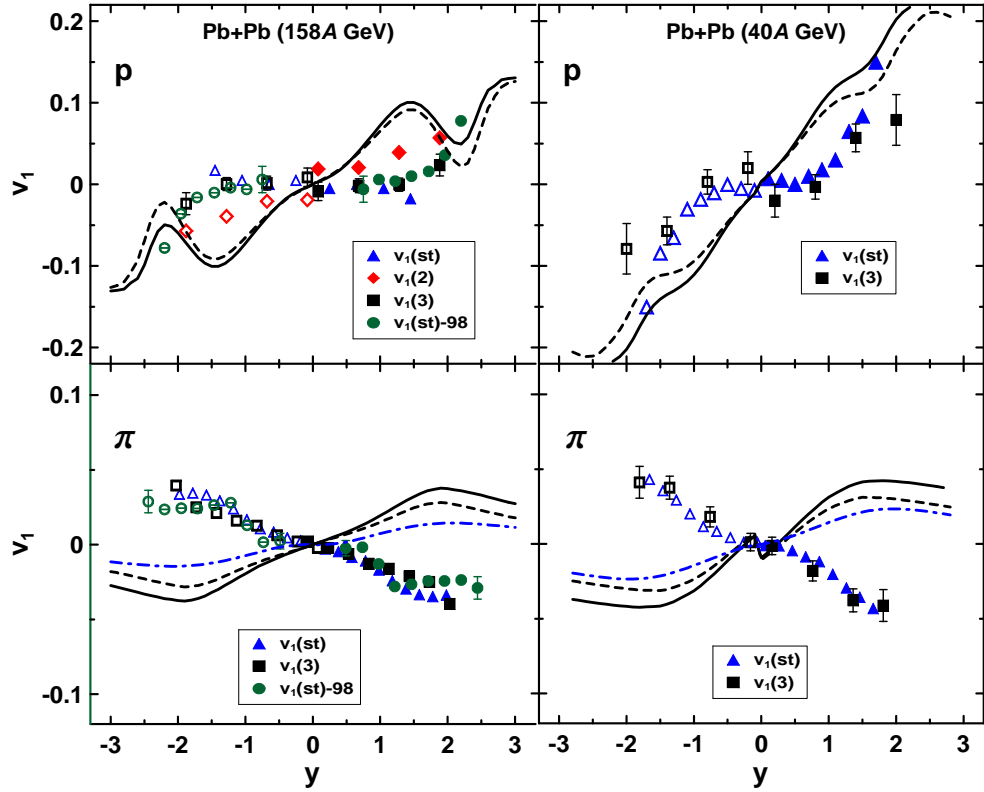


FIG. 13: (Color online) Directed flows of protons (upper panels) and charged pions (lower panels) in semi-central Pb+Pb collisions at $E_{\text{lab}} = 158A$ (right panels) and $40A$ (left panels) GeV as functions of rapidity. The 3-fluid calculations are presented for $b = 5.6$ fm (solid line) and $b = 4$ fm (dashed line). The dot-dashed lines show contributions of the fireball fluid. Experimental data are taken from Ref. [64]. These data were obtained by two different experimental procedures: the standard one ($v_{1,2}(st)$) and the method of n -particle correlations ($v_{1,2}(n)$). Full symbols correspond to measured data, while the open symbols are those reflected with respect to the mid rapidity. Updated data of the NA49 Collaboration [65] ($v_{1,2}(st) - 98$) with acceptance $0.05 < p_T < 0.35$ GeV/c for pions and $0.6 < p_T < 2.0$ GeV/c for protons, are also shown.

mentum conservation, resonance decays, etc. Recently a new method of n -particle correlations has been proposed, which allows to get rid of these non-flow correlations in extracting v_1 and v_2 from genuine azimuthal correlations [67]. Note that only statistical errors are indicated in these figures. The systematic error for protons is 0.005 for v_2 and 0.01 for v_1 at 158A GeV, they can be by 50% larger at 40A GeV [64].

At these energies the 3-fluid v_1 flow significantly differs from the data. Overall, the 3-fluid model predicts an essentially stronger directed flow than that experimentally observed. This disagreement can not be caused by neither uncertainties in the impact parameters nor application of different methods for measuring the directed flow [64]. Moreover, the calculated pion directed flow closely follows the pattern of the proton one, while the pion v_1 data reveal anticorrelation with proton flow. Note that the v_1 flow in the fireball fluid is very weak. Therefore, it is not surprising that pion and proton v_1 are correlated in the 3-fluid model, since they reflect the same collective motion of the baryon-rich fluids.

The reasons for this poor reproduction of v_1 can be two-fold. Certainly the first reason consists in disre-

garding the fact that a part of frozen-out particles is “shadowed” by still hydrodynamically evolving matter (cf. discussion in Subsect. IID). This shadowing means that frozen-out particles cannot freely propagate through the region still occupied by the hydrodynamically evolved matter but rather get reabsorbed into the hydrodynamic phase[109]. Apparently, the baryon directed flow reveals the collective flow of matter, which is mainly built of baryons as the most abundant and heavy component of the system. This collective flow is mainly formed at the early stage of the reaction. Baryon rescatterings within this earlier-formed collective flow at later (freeze-out) stages do not essentially alter the collective transverse momentum of the matter. At the same time, the pions can be much stronger affected by this shadowing, since they are screened by the predominantly baryonic matter, where pions may be essentially decelerated or even absorbed. This can drastically change the pion v_1 .

The second possible reason is very probable from our point of view. It could happen that the hadron EoS used in these simulations is too hard, i.e. results in too

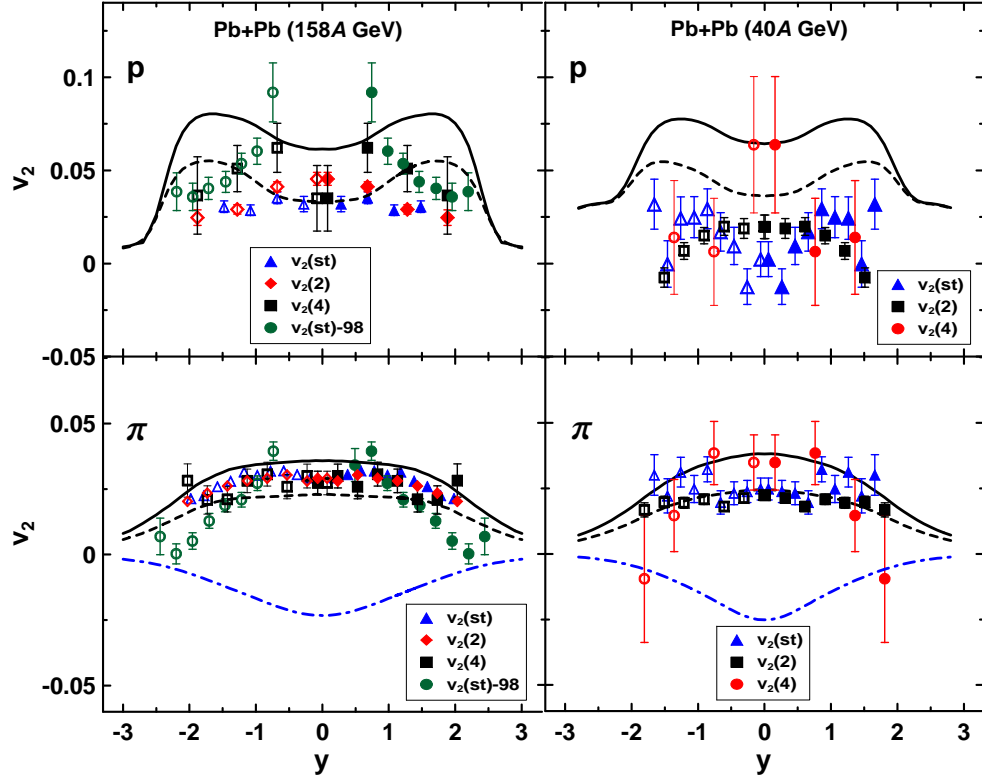


FIG. 14: (Color online) The same as in Fig. 13 but for the elliptic flow.

strong bounce-off of matter. A softer EoS, in particular due to possible phase transition [15, 61], would reduce the strength of the proton directed flow in favor of its better reproduction. Then the problem of correlation/anticorrelation with the pion flow would be more delicate, because the shadowing effect could easily change the sign of the pion flow. An additional argument in favor of a softer EoS is the success of microscopic RQMD [68] and UrQMD [69] models, which correspond to essentially softer EoS, since it is closer to that of the gas. These models qualitatively reproduced early measurements of $v_1(y)$ and $v_2(y)$ for both protons and pions at $E_{\text{lab}} = 158A$ GeV. Recently a good microscopic description of the differential flow at 40A GeV was also obtained [70].

The calculated elliptic flow of protons (see Fig. 14) is positive ("in plane") and somewhat overestimates experimental points at both incident energies, 40A and 158A GeV. Though data at 40A GeV are still controversial, this overestimate again indicates that the used hadronic EoS is too hard. At the same time, the hydrodynamic pion elliptic flow is in surprisingly good agreement with experimental data of the NA49 Collaboration both in magnitude and shape, exhibiting a shallow minimum at the mid rapidity. The elliptic flow v_2 results from the initial spatial asymmetry of non-central nucleus-nucleus collisions. The overlap lens-shaped geometry of two nearly thermalized nuclei is reproduced correctly in the 3-fluid model. As this lens-shaped matter expands, it produces the elliptic flow. Note that mesons emitted from the fire-

ball fluid have negative ("out of plane") v_2 flow.

The earlier pion v_2 data of the NA49 collaboration, taken at smaller acceptance at the top SPS energy [65], have been already analyzed within hydrodynamic approaches. In the expansion model [71] the Bjorken scaling solution [72] was assumed for longitudinal evolution and 2D hydro was solved numerically for transverse one. In this way, the elliptic flow could be estimated only at the mid rapidity point. The full 3D expansion model with postulated initial conditions was applied to the meson elliptic flow by Hirano [73]. In a qualitative agreement with Ref. [71], it was found that ρ -meson decays result in almost vanishing azimuthal anisotropy of pions near the mid rapidity. Let us remind that decays of all relevant resonances are taken into account in our model.

Recently, collective flows in heavy ion collisions from AGS to SPS energies were systematically studied in a transport model with various assumptions on the nuclear mean field [74]. It was found that momentum dependence in the nuclear mean field is of prime importance for the reproduction of the v_1 and v_2 flows. It turned out that generally our results are rather close to those of Ref. [74], if the mean-field momentum dependence is absent. This is precisely the case in our EoS. It is of interest to note that similarly to our results but in contrast with the experiment, the pion directed flow at $E_{\text{lab}} = 158A$ GeV correlates with the proton v_1 independently of the version chosen for the mean field [74].

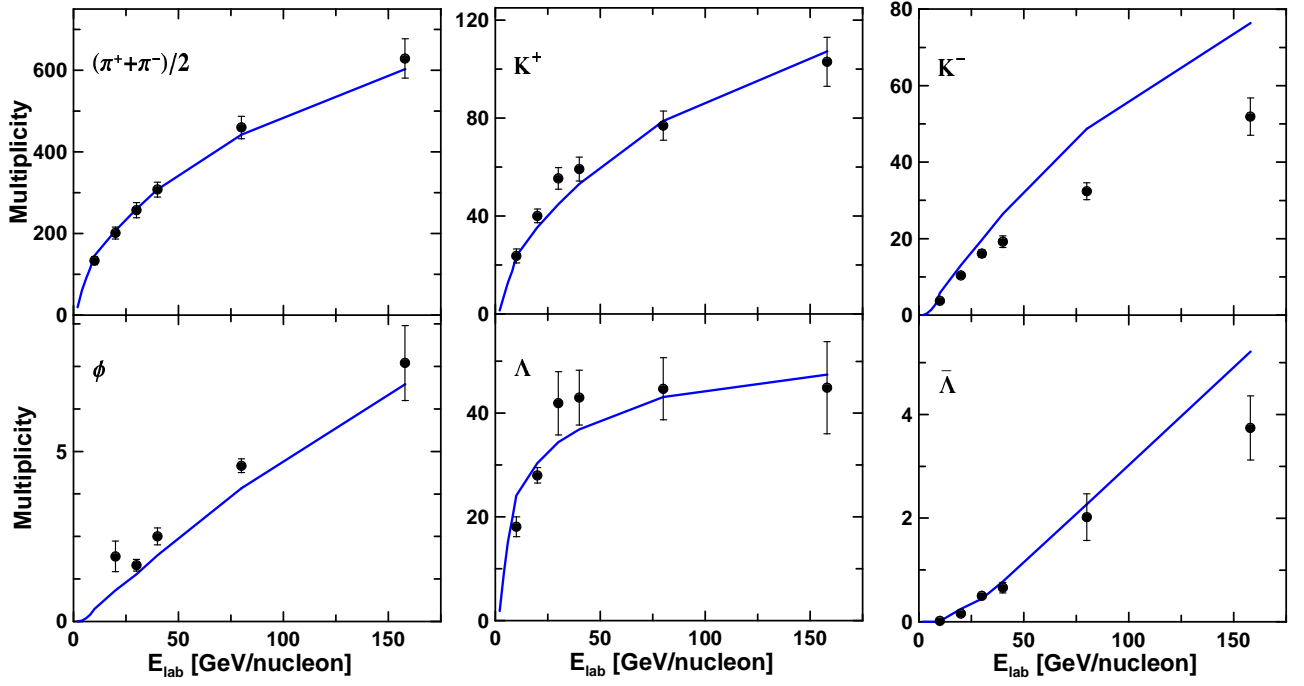


FIG. 15: (Color online) Multiplicities for central Au+Au (at AGS energies) and Pb+Pb (at SPS energies) collisions as functions of the incident energy. Open triangles connected by solid lines represent the 3-fluid results. The experimental data (full circles) are taken from Refs. [58, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85] for SPS energies and from Refs. [86, 87, 88, 89, 90] for AGS energies.

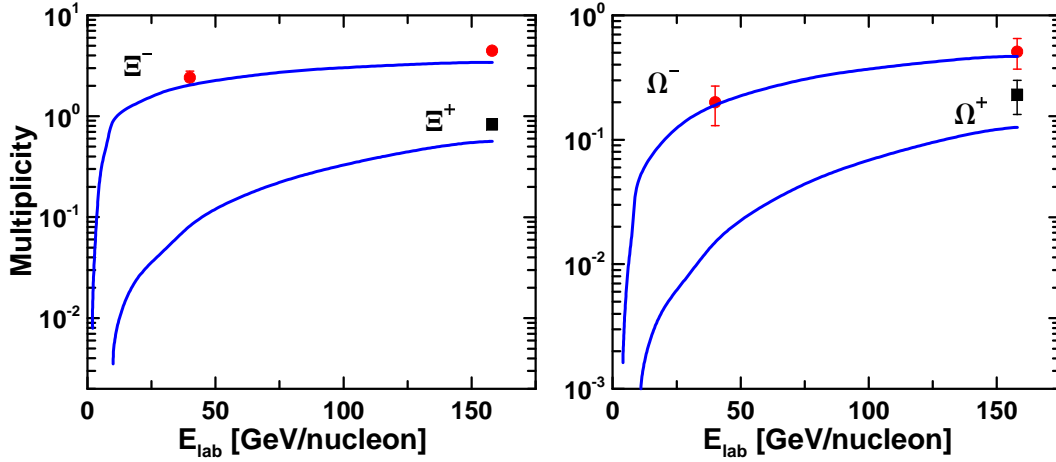


FIG. 16: (Color online) The same as in Fig. 15 but for multiplicities of multi-strange hyperons. The experimental data are taken from Refs. [80, 94, 95]: full circles for hyperons, and full squares for antihyperons.

H. Multiplicities

Though the quality of reproduction of particle multiplicities was already clear from the above presented rapidity distributions, we would like to summarize them in this subsection, as well as present those for hadrons not mentioned above. In Fig. 15, particle multiplicities for central Au+Au (at AGS energies) and Pb+Pb (at SPS energies) collisions as functions of the incident energy are presented and confronted with the experimen-

tal data. The experimental data are taken from Refs. [58, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85] for SPS energies and from Refs. [86, 87, 88, 89, 90] for AGS energies. These data are presented as they were summarized in Ref. [91], where they were scaled to the same 5% experimental trigger. Because of this reduction to the same 5% trigger, all the simulations were performed at the same central impact parameters corresponding to this trigger: $b = 2$ fm for Au+Au and $b = 2.5$ fm for Pb+Pb. Corrections for feed down from weak decays (mostly Ξ^- , Ξ^0

and their antiparticles) were not applied in the original published data. They were, however, estimated to be 6% for Λ and 12% for $\bar{\Lambda}$ [76]. Based on this estimate the Λ and $\bar{\Lambda}$ yields given in Ref. [91] were reduced by 6% and 12%, respectively. This also complies with conditions of our simulations.

We compare our calculated pion multiplicities with the experimental results for half-sum of those for π^+ and π^- , since the model does not distinguish the isospin of particles. Note that the pion multiplicities were fitted by means of the freeze-out parameter and the formation time of the fireball fluid, while all other multiplicities are already predictions of the model. As seen, the model reasonably reproduces the energy dependence of various multiplicities, except that for K^- . The latter is strange, especially in view of that even the rare-channel particles, such as ϕ and $\bar{\Lambda}$, are reasonably reproduced without any additional tuning. Probably, a kind of post-hydro kinetic evolution, similar to that performed in Ref. [22], is required for a proper reproduction of the K^- multiplicity. Indeed, cross sections of the reactions $\bar{K}N \rightarrow \pi\Lambda$ and $\bar{K}N \rightarrow \pi\Sigma$ are very high at low relative momenta [2]. The first reaction transforms a part of K^- into Λ . The resulting Λ enrichment would not contradict to data, since the NA57 data for the Λ production [92] are certainly above the NA49 data presented in Fig. 15. The second reaction results in a loss of strangeness because of the weak decay $\Sigma^\pm \rightarrow N\pi$. In fact, this is the dominant channel for the \bar{K} absorption on nucleons at low energies. In our model such a post-hydro evolution is absent. It is worthwhile to note that the statistical-model fit of the data also overestimate K^- multiplicities at the SPS energies [93].

At comparatively low incident energies $E_{\text{lab}} < 10A$ GeV, the grand canonical treatment of the strangeness production, used in the 3-fluid model, becomes poorly applicable. Therefore, we do not present the 3-fluid predictions at these energies. At energies $E_{\text{lab}} = (30-40)A$ GeV the observed multiplicities of K^+ and Λ are certainly above the smooth curve predicted by the 3-fluid model. This correlates with already above observed fact (see Figs 10 and 14) that agreement with data in this energy region is certainly worse than at other energies.

The excitation functions for production of multi-strange hyperons, Ξ and Ω , and the corresponding antihyperons are presented in Fig. 16. In Ref. [80], Ω^- and Ω^+ were not separated at $E_{\text{lab}} = 40A$ GeV. Due to that their total yield is plotted as Ω^- , since the yield of Ω^+ antihyperons is much lower than that of Ω^- . It is seen that calculated multiplicities of multi-strange hyperons are also in a reasonable agreement with the available data. Note that these yields are strongly underpredicted in the UrQMD model [80].

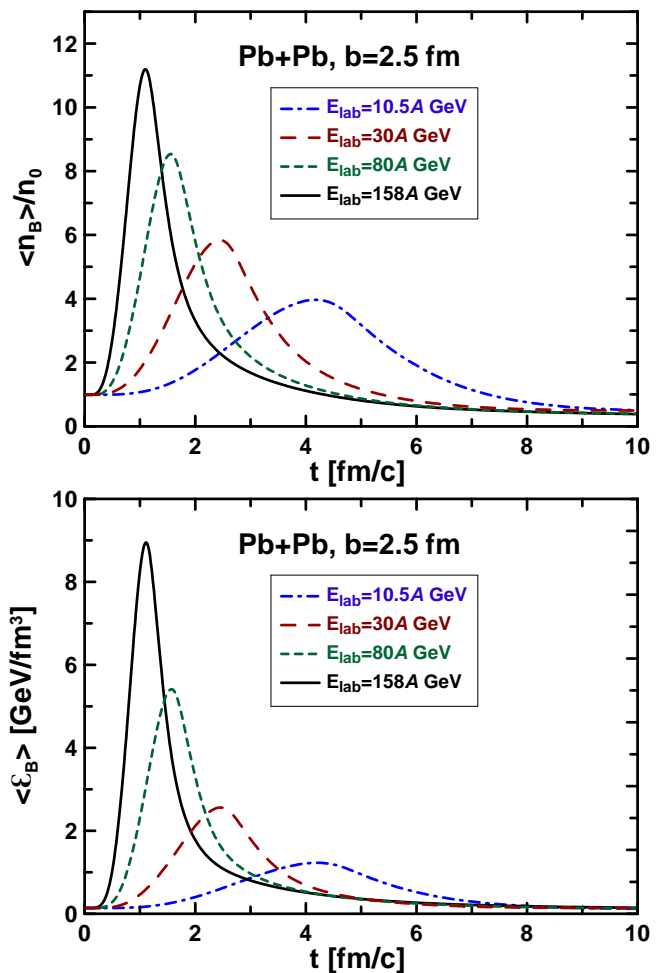


FIG. 17: (Color online) Temporal evolution of average baryon (upper panel) and energy (lower panel) densities for central Pb+Pb collisions within 3-fluid model. The time is counted in the c.m. frame of the colliding nuclei.

IV. GLOBAL EVOLUTION OF NUCLEAR COLLISIONS

In the preceding Section, it was shown that the 3-fluid hydrodynamic model with the simplest, purely hadronic EoS is able to reasonably reproduce a great body of experimental data on relativistic heavy-ion collisions. The considered observables characterize a state of the colliding system at the freeze-out stage. With thus fixed parameters of the model, it is possible to address the question of global evolution of nuclear collisions, e.g., the values of baryon and energy densities achieved in the course of them. An important question also is how long and in which volume these achieved values survive. Dynamics of heavy ion collisions at different bombarding energies is illustrated in Fig. 17. The shown baryon and energy densities are averaged over the whole space occupied by

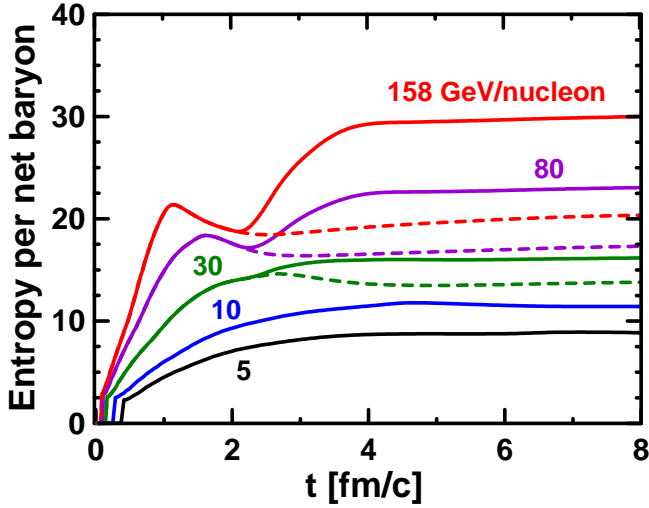


FIG. 18: Time evolution of the entropy per baryon number in Pb+Pb central ($b = 2.5$ fm) collisions at various incident energies E_{lab} . The total entropy is displayed by solid lines, while the entropy of baryon-rich subsystem – by dashed lines.

fluids

$$\langle n_B \rangle = \int d^3x n_B W(x) / \int d^3x W(x), \quad (46)$$

$$\langle \varepsilon \rangle = \int d^3x \varepsilon W(x) / \int d^3x W(x), \quad (47)$$

where ε and n_B are the proper (i.e. in the local rest frame) densities, and the weight function is taken equal to the local proper baryon density $W(x) = n_B(x)$. This weight function was chosen because we are primarily interested here in evolution of dense baryonic matter. The 3-fluid nonequilibrium is quite strong at the initial stage of the collision. The mean values of ε and n_B are calculated by means of averaging these quantities corresponding to either separate fluids (if their mutual stopping has not occurred) or the unified fluid (if the full stopping has happened). To characterize the degree of stopping, we used the auxiliary function introduced in Eq. (23), which reveals quite a sharp transition between transparency ($\vartheta \simeq 1$) and full stopping ($\vartheta \simeq 0$). This convention has been chosen in order to map the nonequilibrium configuration in terms of equilibrium quantities, avoiding unphysical contributions of the Lorentz contraction and collective motion into quantities $\langle \varepsilon \rangle$ and $\langle n_B \rangle$. However, these results at early stages of collisions should be taken with some care because of certain ambiguity of this mapping convention.

As seen from Fig. 17, the average baryon density $\langle n_B \rangle$ increases quickly reaching a maximum, and then the expansion stage comes. The maximal compression of baryonic matter correlates with maximal overlap of colliding nuclei. Evolution of the average energy density $\langle \varepsilon \rangle$ proceeds in a similar way. Maximal values of baryon and energy densities are high enough, exceeding hypothetical threshold for deconfinement phase transition

$\varepsilon \sim 1 \text{ GeV}/\text{fm}^3$ even at $E_{\text{lab}} \sim 10 \text{ A GeV}$. However, these values are slightly lower than those reported in [1], because here averaging over the whole volume of colliding system is carried out rather than over some fixed central region. Note that local peak values may exceed these average values by a factor of about 2.

To get notion of the extent of thermalization achieved in the collision process, in Fig. 18 we show the time evolution of the entropy per baryon number (the latter is conserved quantity) in central collisions. These calculations were performed without freeze-out in order to keep account of the total entropy. We have taken into account the entropy of only formed fireball fluid, since for the unformed part even its definition is ambiguous. Therefore, we deal with an open system, when not all the fireball matter is formed. At the early stage of the collision (till the time instant of complete overlap of nuclei) the entropy quickly rises, which is the evidence of nonequilibrium in the system. After that the total entropy either flattens (at $E_{\text{lab}} < 40 \text{ A GeV}$) or even slightly drops down (at $E_{\text{lab}} > 40 \text{ A GeV}$). This entropy flattening naturally means thermalization. The temporal dropping down of the entropy is related to still continuing production of unformed fireball matter. It starts to form only later, when the entropy starts to rise after the temporal fall. Therefore, at the early stage of the reaction we deal with an open system. The entropy production in the baryon-rich fluids is proportional to the term

$$R = (u_p \cdot u_t) (D_P - D_E) - (D_P + D_E),$$

if the fireball fluid is still unformed, cf. Eqs. (6), (7) and (16). This result follows from the standard derivation [96] of hydrodynamic equation for the entropy flow. Therefore, the fall or rise of the entropy of the baryon-rich fluids depend on the sign of this R . If $R > 0$, the entropy rises. This occurs either at $D_E = 0$, since $(u_p \cdot u_t) \geq 1$, or when $(u_p \cdot u_t) \gg 1$, as it is the case at the initial interpenetration stage. In the region, where the entropy drops down, D_E is still nonzero and $(u_p \cdot u_t)$ is not high enough, therefore $R < 0$. In physical terms, the entropy drops down, when the energy-momentum radiation into the fireball fluid $\propto -[(u_p \cdot u_t) + 1] D_E$ dominates over the internal heating $\propto [(u_p \cdot u_t) - 1] D_P$ of the baryon-rich subsystem.

As seen from Fig. 18, at $E_{\text{lab}} < 40 \text{ A GeV}$ the entropy is approximately conserved already after the colliding nuclei overlap. These overlaps correspond to peak values in Fig. 17 and turning points of the trajectories in Fig. 19. This fact suggests that thermalization in these collisions occurs comparatively early. At higher incident energies, $E_{\text{lab}} > 40 \text{ A GeV}$, the complete thermalization happens at the comparatively late stage ($t \gtrsim 4 \text{ fm/c}$), when the fireball fluid gets formed and the entropy approaches the plateau. However, the entropy of the baryon-rich fluids, displayed by dashed lines in Fig. 18, slowly changes already after the complete overlap of nuclei. It remains constant within 10%. This baryon-rich subsystem can be considered as an approximately thermalized fluid still

interacting with a “bath” of the fireball matter. Therefore, this overlap time can be approximately taken as a time of equilibration in the baryon-rich subsystem at all considered here incident energies.

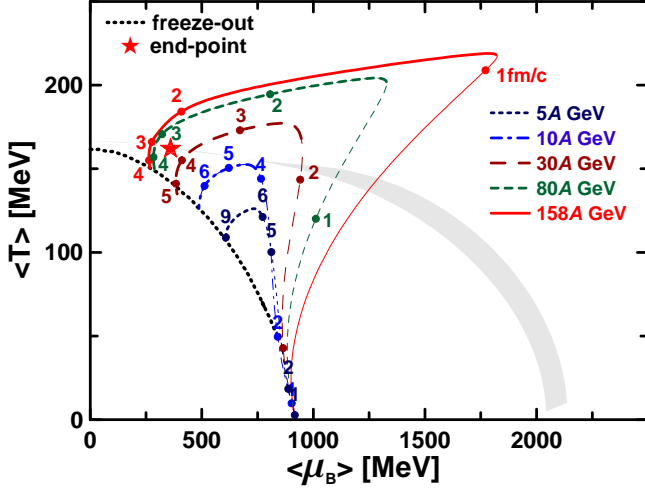


FIG. 19: (Color online) Dynamical trajectories in the (T, μ_B) plane for central Pb+Pb collisions ($b = 2.5$ fm) at various incident energies. Numbers near the dynamical trajectories indicate the evolution time instants in the c.m. frame of the colliding nuclei. Bold parts of trajectories are related to approximately thermalized baryon-rich subsystem, while the thin ones – to yet nonequilibrium evolution. The light-grey shaded region corresponds to the boundary of the phase transition from the hadronic phase to the QGP, as it was estimated in Ref. [97]. Dotted line is the “experimental” freeze-out curve fitted to observed multiplicities in the approximation of the ideal gas model [98] under condition that the energy per hadron is 1 GeV. The star-symbol is the critical end-point calculated in Ref. [6].

Based on the EoS used in simulations, the n_B and ε densities can be recalculated in the temperature–chemical-potential (T, μ_B) representation and displayed in the conventional way as dynamic trajectories in the (T, μ_B) plane, see Fig. 19. In view of the above discussion, we display approximately equilibrium (for the baryon-rich subsystem) parts of the trajectories by bold lines, while those yet nonequilibrium – by thin lines, just in order to keep track of the system evolution. As seen, the compression stage is rather short. For example, at $E_{\text{lab}} = 158A$ GeV the maximal values of $\langle T \rangle$ and $\langle \mu_B \rangle$ are reached during the time of about 1 fm/c. After that a comparatively fast expansion, accompanied by rather slow cooling, starts. At the incident energy of $\approx 30A$ GeV, the dynamical trajectory passes near the critical end-point [6], where the order of the deconfinement phase transition is changed. In fact, the trajectories in the (T, μ_B) plane strongly depend on the used EoS, even if they originate from the same (n_B, ε) initial state. Therefore, this should be taken only as a rough estimate of the relevant incident energy, since the EoS of the hydro

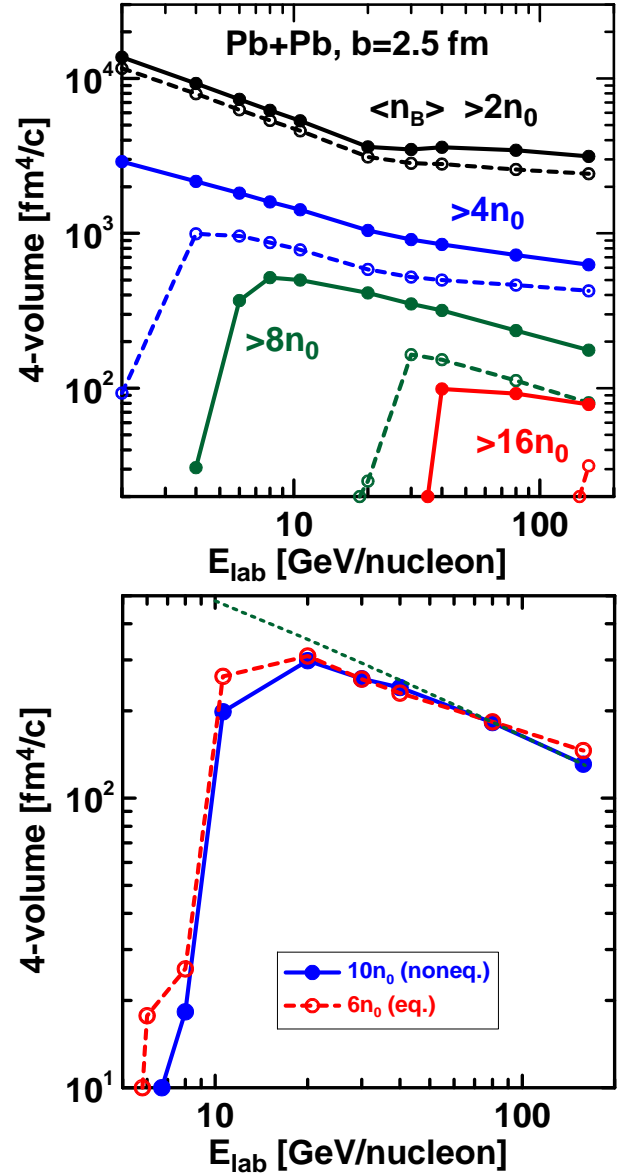


FIG. 20: (Color online) Invariant 4-volume corresponding to conditions on $n_B^{(\text{noneq.})}$ (solid lines) and on $n_B^{(\text{eq.})}$ (long-dashed lines) for central Pb+Pb collisions as function of the incident energy. The dotted line on the lower panel is the 4-volume $\Delta t \cdot \pi R^2 \cdot 2R / \gamma_{cm}$ with $R = 4$ fm, $\Delta t = 3$ fm/c (see the text).

simulations has not involved any QGP phase transition at all. The boundary of the phase transition from the hadronic phase to the QGP, estimated in Ref. [97] (light-grey shaded region Fig. 19), is displayed just to remind on what may happen in this thermodynamic domain, if a EoS involves the phase transition. However, this trajectory representation is not quite informative due to the following reason. In addition to the uncertainties of mapping of nonequilibrium on equilibrium above discussed, the displayed time instants correspond to the cm frame of colliding nuclei and hence are noninvariant. Therefore,

Fig. 19 does not tell us (in invariant way) how long the dense matter survives and in which volume.

The way to overcome the above difficulties was proposed in [1]. It consists in calculation of an invariant 4-volume V_4 in which a quantity q exceeds a given value Q

$$V_4(Q) = \int d^4x \Theta(q - Q). \quad (48)$$

This quantity provides a Lorentz invariant measure of the space-time region, where the quantity q keeps high value $q \geq Q$. In Fig. 20 this 4-volume is shown for two cases: $q = n_B^{(\text{eq.})}$ and $q = n_B^{(\text{noneq.})}$. The first case corresponds to the 4-volume summed only over those regions, where full stopping (i.e. $\vartheta \simeq 0$ in terms of Eq. (23)) has occurred and thermalized n_B exceeds certain value, while in the second case the 4-volume is summed over all regions (i.e. similarly to that in Eq. (46)), including those where stopping has not occurred. As seen, for production of the matter with comparatively high densities ($n_B > 4n_0$), there are certain preferable incident energies, which allow to attain the largest 4-volume. In particular, incident energies (10–40) A GeV, planned at the new GSI facility, are favorable for production of equilibrated matter with baryon densities higher than $6n_0$. This conjecture is heavily based on the hadronic EoS used. However, we expect that it is not too far from the truth, because we fairly well reproduced the observed stopping power, which is of prime relevance to the achieved compression. To get an impression of the scale of the corresponding 4-volume, we draw a short-dashed line presenting the 4-volume $\Delta t \cdot \pi R^2 \cdot 2R / \gamma_{cm}$ composed of Lorentz-contracted cylindrical space volume of radius $R = 4$ fm and time interval $\Delta t = 3$ fm/c, here γ_{cm} is the γ -factor of colliding nuclei in their c.m. frame. It is of interest that the fall-off of the invariant V_4 is defined by this γ -factor.

V. DISCUSSION AND CONCLUSIONS

In this paper we have presented an extension of the 3-fluid model for simulating heavy-ion collisions in the range of incident energies between few and about 200 A GeV. In addition to two baryon-rich fluids, which constitute the 2-fluid model, the baryon-free fireball fluid of newly produced particles with delayed evolution is incorporated. This delay is governed by a formation time, during which the fireball fluid neither thermalizes nor interacts with the baryon-rich fluids. After the formation, it thermalizes and comes into interaction with the baryon-rich fluids. This interaction is estimated from elementary pion-nucleon cross-sections.

The hydrodynamic treatment of heavy-ion collisions is an alternative to kinetic simulations. The hydrodynamic approach has certain advantages and disadvantages. Lacking the microscopic feature of kinetic simulations, it overcomes their basic assumption, i.e. the assumption of binary collisions, which is quite unrealistic in

dense matter. It directly addresses the nuclear EoS that is of prime interest in heavy-ion research. Naturally, we have to pay for these pleasant features of hydrodynamics: the treatment assumes that the non-equilibrium stage of the collision can be described by the 3-fluid approximation.

Taking advantage of the modern computers, substantial work has been also done on improvement of numerics of the model. In particular, it was found that the numerical diffusion of the computation scheme should be as much as possible avoided in order to get reliable results in hydro simulations. To avoid the numerical diffusion the parameters of the scheme should be carefully optimized, as it is described in App. A.

The main unknowns of the present simulations can be briefly summarized as follows: the equation of state and “cross sections”. The EoS is an external input to the calculation and thus can be varied. Our goal is to find an EoS which in the best way reproduces *the largest body of available observables*. The “cross sections” are equally important. They determine friction forces between fluids and hence the nuclear stopping power. At present we have at our disposal only a rough estimate of the friction forces (cf. Ref. [29] for the friction of baryon-rich fluids and Sect. II C for the friction with the fireball). Therefore, we have to fit the friction forces to the stopping power observed in proton rapidity distributions.

In this paper we use a purely hadronic scenario in our simulations. This means a very simple EoS [46], constructed under the requirement that it reproduces saturation of the cold nuclear matter. This EoS is a natural reference point for any other more elaborate EoS. We have found out that the original friction between baryon-rich fluids, incorporated in the model accordingly to estimate of Ref. [29], is evidently insufficient to reproduce observable stopping of nuclear matter. Therefore, the original friction was enhanced by means of the tuning factor ξ_h , cf. Eq. (39), which is plotted in Fig. 2. As seen, the enhancement factor turned out to be unexpectedly large. However, it complies with earlier results of the 2-fluid model with mean mesonic fields [16]. There it was found out that for the proper reproduction of the data on heavy-ion collisions in the energy range from 0.4 A to 0.8 A GeV, where the purely hadronic scenario is certainly applicable, the enhancement by a factor of 3 was required. From this large enhancement we infer that the rough estimate of the friction [24] is really too rough and that further more elaborate microscopic calculations of the friction are required. In doing this at lower energies, we have to rely on pure hadronic effects, e.g. on the concept of the hadron liquid [99] rather than hadron gas. At higher incident energies the friction enhancement could be associated with an indirect manifestation of the onset of the phase transition, although the used EoS does not involve it directly. In Refs. [100, 101, 102] it was argued that just above the critical temperature, the quark-gluon system is still a strongly interacting matter rather than a gas of perturbative partons. This strong interaction

anomalously enhances interaction cross sections in this phase.

With this simple hadronic EoS we have succeeded to reasonably reproduce a great body of experimental data in the incident energy range $E_{\text{lab}} \simeq (1-160)A$ GeV. The list includes proton and pion rapidity distributions, proton transverse-mass spectra, rapidity distributions of Λ and $\bar{\Lambda}$ hyperons and antiprotons, elliptic flow of protons and pions (with the exception of proton v_2 at $40A$ GeV), multiplicities of pions, Λ and $\bar{\Lambda}$ hyperons, K^+ mesons and ϕ mesons, Ξ^\mp and Ω^\mp multistrange hyperons. However, we have also found out certain problems. The calculated yield of K^- is approximately a factor of 1.5 higher than that in the experiment. We have also failed to describe directed transverse flow of protons and pion at $E_{\text{lab}} \geq 40A$ GeV. This especially concerns pions, v_1 of which experimentally anticorrelate with that of protons, while it correlates in our calculations. This failure apparently indicates that the used EoS is too hard and thereby leaves room for a phase transition, which always makes the EoS softer.

The used hadronic EoS allows modifications relating to different incompressibilities (K) of the nuclear matter. We performed simulations with soft ($K = 210$ and 235 MeV) and hard ($K = 380$ MeV) EoS's. We have found out that the data certainly prefer soft EoS. All the calculations presented in this paper correspond to $K = 210$ MeV.

The analysis of the global evolution of central Pb+Pb collisions with present hadronic scenario shows in particular that incident energies $(10-40)A$ GeV, planned at the new GSI facility, are favorable for production of equilibrated matter with baryon densities 6 times higher than the normal nuclear density. This conclusion is heavily based on the simple hadronic EoS used in our calculations. However, we expect that it is not too far from the truth, because we fairly well reproduced the observed stopping power, which is of prime relevance to the achieved compression. Moreover, in this energy range dynamical (T, μ_B) trajectories of nuclear collisions pass in the vicinity of the critical end-point, as it is estimated in Ref. [6]. Note that the presently used EoS does not involve any phase transition. In view of the latter, it is intriguing that discrepancies of the present hadronic scenario with the data (i.e. in the form of the Λ rapidity distribution, proton and pion v_1 and v_2 , and strangeness production) are most clearly seen at incident energies about $40A$ GeV.

Our present experience shows that in order to conclude on the relevance of a particular EoS the whole available set of data should be analyzed in a wide incident energy range with the same fixed parameters of the model. This is one of the main conclusions of our paper. Indeed, any particular piece of data can be apparently fitted by means of fine tuning of these parameters, while to fit the whole set of them, one needs a really good EoS. In the present paper we analyzed various data only fragmentarily, just to fit the parameters and to give an overview of the re-

sulting predictions, since the framework of a single paper does not allow us to do more. Our preliminary conclusion is that the considered hadronic EoS is not perfect at higher incident energies $E_{\text{lab}} \gtrsim 20A$ GeV because of the above mentioned problems. Detailed analysis of particular pieces of available data will be reported in forthcoming papers. We are also going to extend our analysis to other EoS's involving phase transition to the QGP.

Web page

The source code of the 3-fluid model is publically available on the Web page <http://theory.gsi.de/~mfd/>. Free downloads of the code are available from there. Brief instructions on how to run this code are also given on this page.

Acknowledgements

We are grateful to E.L. Bratkovskaya, B. Friman, M. Gadzicki, G.L. Melkumov, I.N. Mishustin, J. Randrup, L.M. Satarov, V.V. Skokov and D.N. Voskresensky for fruitful discussions. We are especially grateful to D.H. Rischke for careful reading of the manuscript and making numerous useful comments. This work was supported in part by the Deutsche Forschungsgemeinschaft (DFG project 436 RUS 113/558/0-3), the Russian Foundation for Basic Research (RFBR grant 06-02-04001 NNIO-a), Russian Minpromnauki (grant NS-1885.2003.2).

APPENDIX A: NUMERICS

The relativistic 3D code for the above described 3-fluid model was constructed by means of modifying the existing 2-fluid 3D code of Refs. [10, 11]. The numeric scheme of the code is based on the modified particle-in-cell method [42, 43], which is an extension of the scheme first applied in Los-Alamos [44]. Taking advantage of modern computers, we have comprehensively tested this numeric scheme in order to find optimal parameters of the calculation.

In the particle-in-cell method the matter is represented by an ensemble of Lagrangian test particles. They are used for calculation of the drift transfer of the baryonic charge, energy and momentum. In the present scheme the test particle has a size of the cell. Therefore, when a single test particle is moved on the grid, it changes quantities in 8 cells, with which it overlaps. These spatially extended particles make the scheme smoother and hence more stable. The transfer due to pressure gradients, friction between fluids and production of the fireball fluid is computed on the fixed grid (so called Euler step of the scheme). Simulation is performed in the frame of equal velocities of colliding nuclei. In view of this combined nature of the numerical scheme there are parameters of the test particles and the grid, which are summarized in

the Table II.

The basic scale of the numerical scheme is chosen along the x direction (the transverse direction in the reaction plane). According to the preset number of cells in the x direction, L_x , the maximal impact parameter b_{\max} , which is foreseen for the simulation, and number cells free of the matter, reserved for transverse expansion, L_x^{free} , the Δx step of the grid is determined as follows

$$\Delta x = \max \{2R_t; R_p + R_t + b_{\max}\} / (L_x - L_x^{\text{free}}),$$

where R_p and R_t are radii of the projectile and target nuclei, respectively[110]. Another transverse step Δy (in the out-of-reaction-plane direction) is taken equal to Δx : $\Delta y = \Delta x$. Δz and Δt are determined by means of preset ratios $\Delta x/\Delta t$ and $\Delta x/\Delta z$. The preset numbers of cells in the y and z directions (L_y and L_z) are used to finally define the size of the grid. The simulation takes place only in the semi-space $y > 0$ because of the symmetry of the system with respect to the reaction plane. The number of baryon-rich test particles ($N_{\text{tot}}^{\text{bar}} = N_{\text{projectile}}^{\text{bar}} + N_{\text{target}}^{\text{bar}}$) is specified at the initialization step of the simulation. As for produced baryon-free fluid, the actual total number of corresponding test particles depends on the incident energy, the size of the cell and the number of produced fireball test particles per cell and per time step (N_f) and is determined only upon the completion of the simulation. The maximal total number of baryon-free test particles $N_{\text{tot}}^{\text{fire}}$ should be larger than the actual total number. $N_{\text{tot}}^{\text{fire}}$ is used to reserve memory for baryon-free test particles.

$E_{\text{lab}}, A \text{ GeV}$	1 ÷ 10	20 ÷ 158*	158**
L_x	150	320	320
L_y	60	130	130
L_z	240	820	820
L_x^{free}	5	43	43
$\Delta x/\Delta t$	3.5	3.5	4
$\Delta x/\Delta z$	1	1	1.2
$N_{\text{tot}}^{\text{bar}}$	$2 \cdot 10^6$	$15 \cdot 10^6$	$15 \cdot 10^6$
N_f	2	2	2
$N_{\text{tot}}^{\text{fire}}$	$3 \cdot 10^6$	$25 \cdot 10^6$	$25 \cdot 10^6$

TABLE II: Basic parameters of the numeric scheme used for simulations of Au+Au and Pb+Pb collisions at various incident energies E_{lab} .

In order to get reasonable accuracy in the simulation, the following basic requirements should be met:

- (i) The grid should be extended enough to prevent escaping the matter beyond this region before it gets frozen out.
- (ii) The grid in the beam (z) direction should be fine enough for a reasonable description of the matter of initially Lorentz-contracted nuclei. From the practical point of view, it is desirable to have more than 30 cells on the Lorentz-contracted nuclear diameter.
- (iii) The well-known Courant-Friedrichs-Lewy criterion

states that the ratios of the space-grid steps to the time step (e.g., $\Delta x/\Delta t$) should be larger than 1 in order to have a consistent and stable algorithm for solving hyperbolic partial-differential equations. To avoid numerical diffusion, this ratios should be taken optimal. As it was found in 1-dimensional simulations of exactly solvable problems [43], the optimal range of these ratios is $2.5 < \Delta x/\Delta t < 6$ with the preferable $\Delta x/\Delta t \simeq 3.5$, minimizing the numerical diffusion. This fact dictates the choice of equal-step grid in all directions ($\Delta x : \Delta y : \Delta z = 1 : 1 : 1$), in spite of Lorentz-contraction of incident nuclei, which is quite strong at high energies. This choice makes the scheme isotropic with respect to the numerical diffusion. However, in view of (i) and (ii) requirements it makes the grid too fine in the transverse directions and thus results in high memory consumption. The need of the equal-step grid in all directions for relativistic hydrodynamic computations within conventional 1-fluid model was pointed out in Ref. [45]. As it was demonstrated there, the matter transport becomes even acausal if this condition is strongly violated.

- (iv) The number of test particles per cell should be high enough to avoid large numerical fluctuations in drift transfer. In practice, this means that this number should be not less than 3 during the whole evolution of expanding matter, till its complete freeze-out.

The final check of these requirements is possible only upon completion of the simulation. The Table II presents the optimized set of parameters for calculations of Au+Au and Pb+Pb collisions at various incident energies. This set was determined by multiple test runs of the code. The reaction Pb + Pb at $E_{\text{lab}} = 158A$ GeV is the most memory and time consuming case considered in simulations. To completely fulfill the requirement (iii) in this calculation we need 8 GB of memory for the central collision (see column 20 ÷ 158* in Table II). In order to comply with memory restrictions, we still had to take slightly unequal steps in different directions: $\Delta x : \Delta y : \Delta z = 1.2 : 1.2 : 1$, $\Delta x/\Delta t = \Delta y/\Delta t = 4$ and $\Delta z/\Delta t = 3.3$ (see column 158** in Table II), for noncentral Pb + Pb collisions at $E_{\text{lab}} = 158A$ GeV.

The freeze-out criterion (30) is checked at the Lagrangian step of the scheme. It is checked in the cells, which overlap with the considered test particle. As mentioned above, the test particle has a volume of the cell. Therefore, each test particle overlap with 8 cells. If the freeze-out criterion is met in *all these cells*, this test particle is frozen out and removed from further hydrodynamic evolution. Thus, the frozen-out test particles are those droplets mentioned in Eq. (33) for the spectrum of observable hadrons. Precisely due to this extended checking in 8 cells, the actual freeze-out energy density[111] ε_{frz} participating in the freeze-out criterion (30) differs from that used in the code $\varepsilon_{\text{frz}}^{\text{code}}$. According to our estimate, the actual freeze-out energy density ε_{frz} is approximately twice as lower than $\varepsilon_{\text{frz}}^{\text{code}}$. This estimate was checked numerically. The reason behind this extended checking in 8 cells is as follows. Had we checked the criterion only

in a single cell, where the test particle is located, we would prevent any hydrodynamic expansion of the system. Indeed, if the matter, during its expansion, starts to fill in a cell, which was empty before that, it occupies less than 1/3 of this cell during the single time step, because the time step should be less than the space step (cf. $\Delta x/\Delta t = 3.5$) for this numerical scheme. Then, all the densities in this newly filled-in cell are quite low at this time step. Therefore, the freeze-out criterion is certainly fulfilled in this cell. Had we confined ourselves to checking only this cell, we would immediately freeze the matter in this cell out, thus preventing hydrodynamic

expansion of the matter.

In order to avoid formation of bubbles of frozen out matter inside the dense environment, we introduced additional criterion of the freeze-out: at least one of the above discussed cells should be empty, i.e. contain no test particles[112]. This additional criterion means that the analyzed test particle is located near the surface of the system, provided, of course, that the number of test particles per cell is large enough (cf. condition (iv)), which excludes origination of such empty cells due to fluctuations.

APPENDIX B: RESONANCE DECAYS

1. General Consideration

Contribution of resonance decays into stable particle spectra has been studied long ago [103] and later analyzed in more detail [104]. To make the paper self-contained, we briefly present here formulas for these decays, which in fact are the same as those in [103, 104] but differ from those in presentation, which, from our point of view, is advantageous for numerical realizations.

Consider a hadronic resonance R with degeneracy factor g_R and a spectral function $A_R(s)$, where s is the resonance mass squared, which decays into n particles

$$R \rightarrow 1 + 2 + \dots + n$$

of masses m_1, m_2, \dots, m_n , through a J th channel of its decay with a branching ratio b_J . Then the distribution of the "1" particle, resulting from this decay, is as follows

$$\begin{aligned} & \left(E_1 \frac{d^3 N_1^{(R \rightarrow 1+2+\dots+n)}}{d^3 p_1} \right)_J = \\ & = b_J \int_{(\sum_{i=1}^n m_i)^2}^{\infty} ds A_R(s) \int \frac{d^3 p_R}{E_R} \left(E_R \frac{d^3 N_R(s, p_R)}{d^3 p_R} \right) \left(E_1 \frac{d^3 \Gamma_1^{(R \rightarrow 1+2+\dots+n)}(s, p_R, p_1)}{d^3 p_1} \right). \end{aligned} \quad (B1)$$

Here, the distribution of the R resonance, produced by the hydrodynamic computation, is

$$E_R \frac{d^3 N_R(s, p_R)}{d^3 p_R} = \frac{g_R V}{(2\pi)^3} \frac{u \cdot p_R}{\exp[\beta(u \cdot p_R - \mu_R)] \pm 1}, \quad (B2)$$

where V is a small proper (i.e. in the rest frame) volume of the fluid element, g_R is degeneracy of the R resonance, u is the hydrodynamic 4-velocity, β is the inverse temperature, μ_R is the chemical potential, p_R is the 4-momentum of the R resonance, $s = p_R^2$, the upper (lower) sign in this expression corresponds to baryonic (mesonic) resonances. The distribution of the "1" particle, resulting from decay of a single R resonance, is expressed as follows

$$E_1 \frac{d^3 \Gamma_1^{(R \rightarrow 1+2+\dots+n)}(s, p_R, p_1)}{d^3 p_1} = \frac{1}{2R_n(s)} \int \left(\prod_{i=2}^n \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(p_R - \sum_{i=1}^n p_i \right) \quad (B3)$$

under assumption that the matrix element of the decay is constant all over the available n -particle phase-space volume [105], which, in its turn, reads

$$R_n(s) = \int \left(\prod_{i=1}^n \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(p_R - \sum_{i=1}^n p_i \right). \quad (B4)$$

In order to obtain the total contribution of the R decay into "1" particle spectrum, we should sum over all J branches of the R decay, in which the "1" particle appears, and take into account that particles identical to "1" can also be among residue particles 2, 3, ..., n

$$E_1 \frac{d^3 N_1^{(R \rightarrow 1+X)}}{d^3 p_1} = \sum_J N_1^J \left(E_1 \frac{d^3 N_1^{(R \rightarrow 1+2+\dots+n)}}{d^3 p_1} \right)_J. \quad (B5)$$

Here N_1^J is the multiplicity of "1" particles in the J th channel of the R decay. Here and below, $E_R = (s + \mathbf{p}_R^2)^{1/2}$ and $E_i = (m_i^2 + \mathbf{p}_i^2)^{1/2}$.

Fortunately, the the major part of integrations involved in Eqs. (B1) and (B3) can be performed analytically. We explicitly present results only for two-particle and three-particle decays, which are dominant for resonances under consideration.

2. Two-Particle Decays

The result for a two-particle decay of a R resonance is as follows

$$\left(E_1 \frac{d^3 N_1^{(R \rightarrow 1+2)}}{d^3 p_1} \right)_J = b_J \frac{g_R V}{(4\pi)^2} \frac{1}{[(u \cdot p_1)^2 - m_1^2]^{1/2}} \int_{(m_1+m_2)^2}^{\infty} ds \frac{A_R(s)}{R_2(s)} \sum_{n=1}^{\infty} (\mp 1)^{n-1} \left[\frac{n\beta E_R + 1}{(n\beta)^2} \exp \{-n\beta(E_R - \mu_R)\} \right]_{E_R^+}^{E_R^-}, \quad (\text{B6})$$

where the upper (lower) sign in this expression corresponds to baryonic (mesonic) resonances,

$$R_2(s) = \frac{\pi}{2s} \lambda^{1/2}(s, m_1^2, m_2^2) \quad (\text{B7})$$

is the 2-particle phase-space volume and

$$E_R^{\pm} = \frac{1}{2m_1^2} \left[(s + m_1^2 - m_2^2) (u \cdot p_1) \pm \lambda^{1/2}(s, m_1^2, m_2^2) [(u \cdot p_1)^2 - m_1^2]^{1/2} \right]. \quad (\text{B8})$$

Here and below

$$\lambda(x, y, z) = (x - y - z)^2 - 4yz \quad (\text{B9})$$

is the standard kinematic function [105]. To arrive to this result, the following expansion

$$\frac{1}{\exp[\beta(u \cdot p_R - \mu_R)] \pm 1} = \sum_{n=1}^{\infty} (\mp 1)^{n-1} \exp \{-n\beta(u \cdot p_R - \mu_R)\} \quad (\text{B10})$$

was used, the advantage of which is that it is rapidly convergent, when $u \cdot p_R > \mu_R$. The opposite limit, i.e. $u \cdot p_R < \mu_R$, corresponds to low temperatures and hence hardly contributes to the resonance production. The $n = 1$ term in this expansion, as well in Eq. (B6), corresponds to the classical Jüttner distribution.

3. Three-Particle Decays

The result for a three-particle decay of a R resonance is as follows

$$\begin{aligned} \left(E_1 \frac{d^3 N_1^{(R \rightarrow 1+2+3)}}{d^3 p_1} \right)_J &= b_J \frac{g_R V}{32\pi} \frac{1}{[(u \cdot p_1)^2 - m_1^2]^{1/2}} \int_{(m_1+m_2+m_3)^2}^{\infty} ds \frac{A_R(s)}{R_3(s)} \\ &\times \int_{\sqrt{s}}^{\infty} dE_R \theta(X^+/X^-) \\ &\times \theta \left(\frac{(m_2^2 - m_3^2)^2 - (m_2^2 + m_3^2) X^+ + |m_2^2 - m_3^2| \sqrt{\lambda^+}}{(m_2^2 - m_3^2)^2 - (m_2^2 + m_3^2) X^- + |m_2^2 - m_3^2| \sqrt{\lambda^-}} \right) \theta \left(\frac{-(m_2^2 + m_3^2) + X^+ + \sqrt{\lambda^+}}{-(m_2^2 + m_3^2) + X^- + \sqrt{\lambda^-}} \right) \\ &\times \left[\left(\sqrt{\lambda^+} - \sqrt{\lambda^-} \right) + |m_2^2 - m_3^2| \ln \frac{X^+}{X^-} \right. \\ &- |m_2^2 - m_3^2| \ln \frac{(m_2^2 - m_3^2)^2 - (m_2^2 + m_3^2) X^+ + |m_2^2 - m_3^2| \sqrt{\lambda^+}}{(m_2^2 - m_3^2)^2 - (m_2^2 + m_3^2) X^- + |m_2^2 - m_3^2| \sqrt{\lambda^-}} \\ &\left. - (m_2^2 + m_3^2) \ln \frac{-(m_2^2 + m_3^2) + X^+ + \sqrt{\lambda^+}}{-(m_2^2 + m_3^2) + X^- + \sqrt{\lambda^-}} \right] \frac{E_R}{\exp[\beta(E_R - \mu_R)] \pm 1} \end{aligned} \quad (\text{B11})$$

where the upper (lower) sign in this expression again corresponds to baryonic (mesonic) resonances,

$$X^+ = \min(\tilde{X}^+, X_{max}), \quad X^- = \max(\tilde{X}^-, X_{min}), \quad (B12)$$

$$X_{max} = (s^{1/2} - m_1)^2, \quad X_{min} = (m_2 + m_3)^2, \quad (B13)$$

$$\tilde{X}^\pm = s + m_1^2 - 2E_R(u \cdot p_1) \pm 2(E_R^2 - s)^{1/2}[(u \cdot p_1)^2 - m_1^2]^{1/2}, \quad (B14)$$

$$\lambda^\pm = \lambda(X^\pm, m_2^2, m_3^2), \quad (B15)$$

$$R_3(s) = \left(\frac{\pi}{2}\right)^2 \frac{1}{s} \int_{X_{min}}^{X_{max}} \frac{dX}{X} \lambda^{1/2}(s, X, m_1^2) \lambda^{1/2}(X, m_2^2, m_3^2), \quad (B16)$$

$\theta(\lambda^+)$, $\theta(\lambda^-)$, etc. are step functions which define the accessible kinematic region.

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- [1] B.Friman, W. Nörenberg, and V.D. Toneev, Eur. Phys. J. **A3**, 165 (1998).
- [2] W. Cassing and E.L. Bratkovskaya, Phys. Rep. **308**, 65 (1999).
- [3] P. Braun-Munzinger, J. Cleymans, H. Oeschler, and K. Redlich, Nucl. Phys. **A697**, 902 (2002).
- [4] (M. Gazdzicki *et al.*, (NA49 Collaboration) J. Phys. **G30**, S701 (2004).
- [5] Conceptual Design Report “*An International Accelerator Facility for Beams of Ions and Antiprotons*”, <http://www.gsi.de/GSI-Future/cdr/>
- [6] Z. Fodor and S.D. Katz, JHEP **203**, 14 (2002); JHEP **404**, 50 (2004).
- [7] M. Stephanov, K. Rajagopal and E. Shuryak, Phys. Rev. D **60**, 114028 (1999).
- [8] A.A. Amsden, A.S. Goldhaber, F.H. Harlow, and J.R. Nix, Phys. Rev. C **17**, 2080 (1978).
- [9] R.B. Clare and D. Strottman, Phys.Rept. **141** 177 (1986).
- [10] I.N. Mishustin, V.N. Russkikh, and L.M. Satarov, Yad. Fiz. **48**, 711 (1988) [Sov. J. Nucl. Phys. **48**, 454 (1988)]; Nucl. Phys. **A494**, 595 (1989).
- [11] I.N. Mishustin, V.N. Russkikh, and L.M. Satarov, Yad. Fiz. **54**, 429 (1991) [Sov. J. Nucl. Phys. **54**, 260 (1991)]; in *Relativistic heavy ion physics*, L.P. Csernai and D.D. Strottman (eds), (World Scientific, 1991) p.179.
- [12] Yu.B. Ivanov, E.G. Nikonov, W. Nörenberg, V.D. Toneev, and A.A. Shandenko, Heavy Ion Phys. **15**, 127 (2002).
- [13] U. Katscher, D.H. Rischke, J.A. Maruhn, W. Greiner, I.N. Mishustin, and L.M. Satarov, Z. Phys. **A346**, 209 (1993); U. Katscher, J.A. Maruhn, W. Greiner, and I.N. Mishustin, Z. Phys. **A346**, 251 (1993); A. Dumitru, U. Katscher, J.A. Maruhn, H. Stöcker, W. Greiner, and D.H. Rischke, Phys. Rev. C **51**, 2166 (1995); Z. Phys. **A353**, 187 (1995).
- [14] J. Brachmann, A. Dumitru, J.A. Maruhn, H. Stöcker, W. Greiner, and D.H. Rischke, Nucl. Phys. **A619**, 391 (1997); A. Dumitru, J. Brachmann, M. Bleicher, J.A. Maruhn, H. Stöcker, and W. Greiner, Heavy Ion Phys. **5**, 357 (1997); M. Reiter, A. Dumitru, J. Brachmann, J.A. Maruhn, H. Stöcker, and W. Greiner, Nucl. Phys. **A643**, 99 (1998); M. Bleicher, M. Reiter, A. Dumitru, J. Brachmann, C. Spieles, S.A. Bass, H. Stöcker, and W. Greiner, Phys. Rev. C **59**, R1844 (1999); J. Brachmann, A. Dumitru, H. Stöcker, and W. Greiner, Eur. Phys. J. **A8**, 549 (2000);
- [15] J. Brachmann, S. Soff, A. Dumitru, H. Stöcker, J.A. Maruhn, W. Greiner, L.V. Bravina, and D.H. Rischke, Phys. Rev. C **61**, 024909 (2000).
- [16] V. N. Russkikh, Yu. B. Ivanov, Yu. E. Pokrovsky, and P. A. Henning, Nucl. Phys. **A572**, 749 (1994).
- [17] Yu. B. Ivanov, V. N. Russkikh, M. Schönhofen, M. Cubero, B. L. Friman, and W. Nörenberg, Z. Phys. **A340**, 385 (1991); Yu. B. Ivanov and V. N. Russkikh, Nucl. Phys. **A580**, 614 (1994); Yad. Fiz. **58**, 2180 (1995) [Phys. of At. Nucl. **58**, 2066 (1995)]; Yad. Fiz. **60**, 743 (1997) [Phys. of At. Nucl. **60**, 660 (1997)]; V. N. Russkikh and Yu. B. Ivanov, Nucl. Phys. **A591**, 699 (1995); S. Ayik, Yu. B. Ivanov, V. N. Russkikh, and W. Nörenberg, Nucl. Phys. **A578**, 640 (1994).
- [18] Yu.B. Ivanov, I.N. Mishustin, and L.M. Satarov, Nucl. Phys. **A433**, 713 (1985).
- [19] Yu.B. Ivanov and L.M. Satarov, Nucl. Phys. **A446**, 727 (1985).
- [20] Y. Hama, R. Andrade, F. Grassi, O. Socolowski, T. Kodama, B. Tavares, and S.S. Padula, hep-ph/0510101; V.V. Skokov and V.D. Toneev, nucl-th/0509085, nucl-th/0601160.
- [21] J. Sollfrank, P. Huovinen, M. Kataja, P.V. Ruuskanen, M. Prakash, and R. Venugopalan, Phys. Rev. C **55**, 392 (1997); P. Huovinen, P.V. Ruuskanen and J. Sollfrank, Nucl. Phys. **A650**, 227 (1999); P.F. Kolb, J. Sollfrank, P.V. Ruuskanen, and U. Heinz, Nucl. Phys. **A661**, 349 (1999); C. Nonaka and S.A. Bass, nucl-th/0510038.
- [22] C.M. Hung and E.V. Shuryak, Phys. Rev. Lett. **75**, 4003 (1995); C.M. Hung and E. Shuryak, Phys. Rev. C **57**, 1891 (1998); D. Teaney, J. Lauret, and E.V. Shuryak, nucl-th/0110037.
- [23] V.N. Russkikh, Yu.B. Ivanov, E.G. Nikonov, W. Nörenberg, and V.D. Toneev, Phys. Atom. Nucl. **67**, 199 (2004); V.D. Toneev, Yu.B. Ivanov, E.G. Nikonov, W. Nörenberg, and V.N. Russkikh, Phys. of Part. and Nucl. Lett. **2**, 288 (2005).
- [24] S. Bass, *et al.*, Prog. Part. Nucl. Phys. **41**, 225 (1998).
- [25] W. Cassing and E.L. Bratkovskaya, Phys. Rep. **308**, 65

- (1999).
- [26] L. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994); *ibid.* **49**, 3352 (1994); F. Gelis, K. Kajantie, and T. Lappi, Phys. Rev. C **71**, 024904 (2005).
 - [27] I.N. Mishustin and J. I. Kapusta, Phys. Rev. Lett. **88**, 112501 (2002); V.K. Magas, L.P. Csernai, and D. Strottman, Nucl. Phys. **A712**, 167 (2002).
 - [28] Yu.B. Ivanov, Nucl. Phys. **A474**, 669 (1987).
 - [29] L.M. Satarov, Yad. Fiz. **52**, 412 (1990) [Sov. J. Nucl. Phys. **52**, 264 (1990)].
 - [30] Madappa Prakash, Manju Prakash, R. Venugopalan, and G. Welke, Phys. Rep. **227**, 321 (1993).
 - [31] G.A. Milekhin, Zh. Eksp. Teor. Fiz. **35**, 1185 (1958); Sov. Phys. JETP **35**, 829 (1959); Trudy FIAN **16**, 51 (1961).
 - [32] F. Cooper and G. Frye, Phys. Rev. D **10**, 186 (1974).
 - [33] K.A. Bugaev, Nucl. Phys. **A606**, 559 (1996).
 - [34] J.J. Neumann, B. Lavrenchuk, and G. Fai, Heavy Ion Physics **5**, 27 (1997).
 - [35] L.P. Csernai, Z. Lázár, and D. Molnár, Heavy Ion Physics **5**, 467 (1997).
 - [36] K.A. Bugaev and M.I. Gorenstein, nucl-th/9903072; K.A. Bugaev, M.I. Gorenstein, and W. Greiner, J.Phys.**G25**, 2147 (1999); Heavy Ion Physics **10**, 333 (1999).
 - [37] Cs. Anderlik, L.P. Csernai, F. Grassi, Y. Hama, T. Kodama, Zs. Lázár, and H. Stöcker, Heavy Ion Physics **9**, 193 (1999).
 - [38] K. Tamošiūnas and L.P. Csernai, Eur.Phys.J. **A20**, 269 (2004).
 - [39] E. Molnar, L. P. Csernai, V. K. Magas, A. Nyiri, and K. Tamošiūnas, nucl-th/0503047; Authors: E. Molnar, L. P. Csernai, V. K. Magas, Zs. I. Lazar, A. Nyiri, and K. Tamošiūnas, nucl-th/0503048.
 - [40] F. Grassi, Y. Hama, and T. Kodama, Phys. Lett. **B355**, 9 (1995); Z. Phys. **C73**, 153 (1996); Yu.M. Sinyukov, S.V. Akkelin, and Y. Hama, Phys. Rev. Lett. **89**, 052301 (2002); Y. Hama, T. Kodama, and O. Socolowski, Braz. J. Phys. **35**, 24 (2005).
 - [41] F. Grassi, Braz. J. Phys. **35**, 52 (2005).
 - [42] A.S. Roshal and V.N. Russkikh, Yad. Fiz. **33**, 1520 (1981).
 - [43] V.N. Russkikh, in “Numerical Methods of Medium Mechanics” (in Russian), Novosibirsk, vol. **1(18)** 104 (1987).
 - [44] F.H. Harlow, A.A. Amsden, and J.R. Nix, J. Comp. Phys. **20**, 119 (1976).
 - [45] B. Waldhauser, D.H. Rischke, U. Katscher, J.A. Maruhn, H. Stöcker, and W. Greiner, Z. Phys. C **54**, 459 (1992).
 - [46] V.M. Galitsky and I.N. Mishustin, Sov. J. Nucl. Phys. **29**, 181 (1979).
 - [47] P. Danielewicz, R. Lacey, and W.G. Lynch, Science **298**, 1592 (2002); P. Danielewicz, nucl-th/0512009.
 - [48] H. Weber, E.L. Bratkovskaya, W. Cassing, and H. Stöcker, Phys. Rev. C **67**, 014904 (2003).
 - [49] L. Ahle *et al.* (E802 Collab.), Phys. Rev. C **60**, 064901 (1999).
 - [50] J. Barrette *et al.* (E877 Collab.), Phys. Rev. C **62**, 024901 (2000).
 - [51] B.B. Back *et al.*, (E917 Collab), Phys. Rev. Lett. **86**, 1970 (2001).
 - [52] H. Appelshäuser *et al.* (NA49 Collab.), Phys. Rev. Lett. **82**, 2471 (1999).
 - [53] G. E. Cooper (for the NA49 Collab.) Nucl. Phys. A **661**, 362c (1999).
 - [54] T. Anticic *et al.* (NA49 Collab.), Phys. Rev. C **69**, 024902 (2004).
 - [55] N.Herrmann, Nucl.Phys. **A610**, 49c (1996).
 - [56] J. L. Klay *et al.* (E895 Collab.), Phys. Rev. C **68**, 054905 (2003).
 - [57] M. van Leeuwenet *et al.* (NA49 collab.), Nucl. Phys. **A715**, 161c (1999).
 - [58] S. V. Afanasiev *et al.* (NA49 Collab.), Phys. Rev. C **66**, 054902 (2002).
 - [59] T. Anticic *et al.* (NA49 Collab.), Phys.Rev.Lett. **93**, 022302 (2004).
 - [60] G.I. Veres *et al.* (NA49 Collab.), Nucl. Phys. **A661**, 383 (1999).
 - [61] D.H. Rischke, Y. Pürsün, J.A. Maruhn, H. Stöcker, and W. Greiner, Heavy Ion Phys. **1**, 309 (1996); D.H. Rischke, Nucl. Phys. **A610**, 88c (1996).
 - [62] P. Danielewicz and G. Odyniec, Phys. Lett. B **157**, 146 (1985).
 - [63] J. Barrette *et al.* (E877 Collab.), Phys. Rev. C **56**, 3254 (1997).
 - [64] C. Alt *et al.* (NA49 Collab.), Phys. Rev. C **68**, 034903 (2003).
 - [65] H. Appelshäuser *et al.* (NA49 Collab.), Phys. Rev. Lett. **80**, 4136 (1998).
 - [66] A.M. Poskanzer and S.A. Voloshin, Phys. Rev. C **58**, 1671 (1998).
 - [67] N. Borghini, P.M. Dinh, and J.Y. Ollitrault, Phys. Rev. C **63**, 054906 (2001); Phys. Rev. C **64**, 054901 (2001); Phys. Rev. C **66**, 014905 (2002).
 - [68] H. Liu, S. Panitkin, and N. Xu, Phys. Rev. C **59**, 348 (1999).
 - [69] S. Soff, S.A. Bass, M. Bleicher, H. Stoecker, and W. Greiner, nucl-th/9903061.
 - [70] H. Stöcker, E.L. Bratkovskaya, M. Bleicher, S. Soff, and Z. Zhu, J. Phys. **G31**, S929 (2005).
 - [71] J.P. Kolb, J. Sollfrank, and U. Heinz, Phys. Lett. **B 459**, 667 (1999).
 - [72] J.D. Bjorken, Phys. Rev. D **27**, 140 (1983).
 - [73] T. Hirano, Phys. Rev. Lett. **86**, 2754 (2001).
 - [74] M. Isse, A. Ohnishi, N. Otuka, P.K. Sahu, and Y. Nara, Phys. Rev. C **72**, 064908 (2005).
 - [75] A. Mischke *et al.*, Nucl. Phys. A **715**, 453 (2003).
 - [76] T. Anticic *et al.* [NA49 Collaboration], Phys. Rev. Lett. **93**, 022302 (2004).
 - [77] V. Friese *et al.* (NA49 Collaboration), Nucl. Phys. **A698**, 487 (2002).
 - [78] S.V. Afanasiev *et al.* (NA49 Collaboration), Phys. Lett. **B538**, 275 (2002).
 - [79] S. V. Afanasev *et al.* (NA49 Collaboration), Phys. Lett. **B491**, 59 (2000).
 - [80] C. Alt *et al.* (NA49 Collaboration), Phys. Rev. Lett. **94**, 192301 (2005).
 - [81] C. Alt *et al.* (The NA49 Collaboration), J. Phys. G **30**, S119 (2004).
 - [82] M. Gazdzicki *et al.* (NA49 Collaboration), J. Phys. G **30**, S701 (2004).
 - [83] C. Meurer (NA49 Collaboration), J. Phys. G **30**, S1325 (2004).
 - [84] A. Richard (NA49 Collaboration), Hot Quarks, Taos, New Mexico, July 2004.
 - [85] C. Blume (NA49 Collaboration), SQM2004, Cape Town, September 2004, to appear in J. Phys. G.

- [86] L. Ahle *et al.*, (E802 Collaboration), Phys. Rev. **C59**, 2173 (1999).
- [87] L. Ahle *et al.*, (E802 Collaboration), Phys. Rev. C **60**, 044904 (1999).
- [88] L. Ahle *et al.*, (E802 Collaboration), Phys. Rev. C **60**, 064901 (1999).
- [89] S. Albergo *et al.*, Phys. Rev. Lett. **88**, 062301 (2002).
- [90] S. Ahmad *et al.*, Phys. Lett. **B382**, 35 (1996).
- [91] F. Becattini, M. Gazdzicki, A. Keranen, J. Manninen, and R. Stock, Phys. Rev. C **69**, 024905 (2004).
- [92] F. Antinori *et al.*, (NA57 Collaboration), Phys. Lett. **B595**, 68 (2004); J. Phys. **G31**, 1345 (2005).
- [93] A. Andronic, P. Braun-Munzinger, and J. Stachel, nucl-th/0511071.
- [94] S.V. Afanasiev *et al.* (NA49 Collaboration), Phys. Lett. **B538**, 275 (2002).
- [95] C. Meurer (NA49 Collaboration), J. Phys. G **30**, S1325 (2004).
- [96] L.D. Landau and E.M. Lifshitz, “*Fluid Mechanics*”, Pergamon Press, Oxford, 1979.
- [97] V.D. Toneev, E.G. Nikonov, B. Friman, W. Nörenberg, and K. Redlich, Eur. Phys. J. **C32**, 399 (2004).
- [98] J. Cleymans and K. Redlich, Phys. Rev. Lett. **81** 5284 (1998).
- [99] D.N. Voskresensky, Nucl. Phys. **A774**, 378 (2004).
- [100] E.V. Shuryak and I. Zahed, Phys. Rev. C **70**, 021901(R) (2004).
- [101] G.E. Brown, Ch.-H. Lee, and M. Rho, hep-ph/0402207.
- [102] A. Peshier and W. Cassing, Phys. Rev. Lett. **94**, 172301 (2005).
- [103] J.I. Kapusta, Phys. Rev. C **16**, 1493 (1977).
- [104] J. Sollfrank, P. Koch, and U. Heinz, Z. Phys. **C52**, 593 (1991).
- [105] E. Byckling and K. Kajantie, *Particle Kinematics*, John Wiley and Sons, London 1773.
- [106] Note that in the mean-field multi-fluid dynamics [16, 17] this problem is overcome.
- [107] In fact, the particle emission would create pressure against the emitting surface, if we allow feedback between the frozen-out matter and the still hydrodynamic system, i.e. if some particles from the frozen-out matter return into the hydrodynamic system. This is a long-standing problem of the freeze-out procedure still waiting for its consistent solution.
- [108] Note that Σ^0 and $\bar{\Sigma}^0$ suffer the sequence of electromagnetic and weak decays: $\Sigma^0 \rightarrow \Lambda \gamma \rightarrow N \pi \gamma$.
- [109] Of course, the conventional shadowing (of participants by spectators) is automatically taken into account in our calculations: there simply is no emission from our participants into the direction of spectators, since they are not separated by any freeze-out region. Moreover, the “participants” and “spectators” are very approximate terms for different parts of the unified hydrodynamic system.
- [110] Here, we assumed that, if colliding nuclei are different, then the larger one is target.
- [111] related to the cell, where the center coordinate of the frozen-out test particle belongs to
- [112] By definition, the cell is empty, if no center coordinate of any test particle belongs to this cell.